

CONGESTION PROBLEMS IN FIELD ARTILLERY  
OPERATIONS

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Monterey, California



## THESIS

CONGESTION PROBLEMS IN FIELD ARTILLERY  
OPERATIONS

by

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SEPTEMBER 1976

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by

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## ABSTRACT

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## I. INTRODUCTION

As the field of Operations Research has grown tremendously since World War II, one of the most important trends has been to adopt ever more complex techniques of modeling combat for military analysis purposes. Unfortunately, as our models become more and more complex, it is harder to recognize the impact which various assumptions have on the outcomes of our computer simulations and other studies. Indeed, we soon begin to feel that we are losing sight of the forest because of the presence of all those trees, whose details we are concentrating on to the exclusion of other, major factors which may also have vast tactical implications. While this problem has recently been recognized by the military analysis community, it is not really new. As long ago as the 1870's, the international war gaming community was rocked by the great debate between the Rigid and Free Kriegspielers as to which aspects of war games should have precedence: a strict interpretation of tried and tested rules, or the free play of military experience and its associated ease of player understanding.

However, it is not the purpose of this paper to trace the historical development of this controversy (interesting as it is) and its effects on the modern version of the problem. Rather, the present task is concerned with cutting through some of that maze of detail in many military models, in an attempt to obtain some valid information about the effects of different major assumptions on the outcomes of the various stochastic simulations often employed in cost-effectiveness studies and the implications they hold



for the analysis of weapons systems.

Because of the impact of the 1973 Mideast war on major military doctrine, particularly the employment of Field Artillery systems, I have chosen to scale down the magnitude of the current task to a specific examination of the Direct Support fire support system and the implications our assumptions have on the capabilities of what is currently called the "dedicated" battery. As a result of the following analysis, closed-form solutions will be obtained which will allow us to measure the effects of the different distributional assumptions used in various stochastic models (such as DYTACS) when attempting to portray the randomness of combat due to variations in human and weapon response times. These equations will then assist us in answering a current, hotly debated question: which models are better for analyzing combat, stochastic or deterministic ones? However, before leaping to our final conclusions, it is necessary to present some background material which illustrates the importance of this investigation and which also sheds some light on the nature of the fire support system we will be analyzing.





## II. BACKGROUND

Since Napoleon's innovation of employing artillery in direct support of particular line units, the tactical concepts of field artillery employment have changed little over the years although improving technology has enabled weapons to be fired at longer ranges with greater lethality and, with the advent of modern communications systems, in an indirect manner rather than remaining limited to line of sight operations. However, even though artillery fire support has been valued highly by combat soldiers, relatively recent developments in warfare, particularly the introduction of air power and the tank, have tended to overshadow the importance of artillery on the battlefield since these newer weapons and techniques have been more successful in capturing the imagination of both the general and military public. This particular fascination came to the fore most prominently with the German use of the "blitzkrieg" in World War II and has remained in current use particularly in the Arab-Israeli conflicts of 1956 and 1967. These short wars were significant in reinforcing the current emphasis on armored formations and the use of air power since the employment of small but disciplined and well-armed forces against a vastly numerically superior enemy highlighted the importance of shock and firepower on the battlefield and served to confirm in the minds of many that the combination of armor and air support was virtually unbeatable.

As is natural in international affairs, when one party to a conflict develops a technique or a particular weapon which gives it superiority, the opposing party is soon



forced to seek countermeasures to improve its own position and avoid repeated defeat. The conflict in the Middle East was no exception. Thus, the Arabs, with Soviet advice and technical assistance, sought to find alternatives which would enable them to neutralize or significantly reduce the Israeli advantages in armor and air power. The degree to which they achieved their goals is more than amply illustrated in the battles of the Yom Kippur War of October 1973, which led Major General Chaim Herzog to write:

...the comfortable feeling that Israeli air power provided an answer to overwhelming Arab preponderance in artillery soothed any sense of urgency about building Israel's artillery strength. For years the Israeli Command deluded itself into believing that air power was the answer to the problem of the country's weakness in artillery - hence the very unrealistic ratio of forces in artillery during the Yom Kippur War. [Ref. 11, p. 252-253]

and

To a degree air power will obviously not be as influential as it has been and will affect the battlefield less than it did. The proliferation of light, portable missile launchers in the front line means that close support will be the exception to the rule in the future, with the air force being obliged to concentrate on isolating the field of battle, maintaining supremacy in the air and destroying the forces in and near the field of battle[Ref. 11, p. 261].

In analyzing the import of these statements for future conflicts we must keep two important facts in mind:

1. This most recent war was fought with almost the entire arsenal of modern weapons, with the exception of nuclear ammunition.
2. The current doctrine of both NATO and the Soviet-bloc forces was exercised by the participating armies.



In view of the above, it is apparent that the lessons to be learned from the October war are of vital interest not only to those nations in the Middle East who figure to be possible future combatants but also to the NATO forces which currently embrace many of the procedures which were employed by the Israel Defense Forces (IDF) during that war. In particular, the effect of reduced air power influence is of critical importance to the United States with our current tactical emphasis on both close air support and helicopter operations. Since the helicopter is even more vulnerable to conventional anti-aircraft fire and radar guided missiles than swift jets, Major General Herzog's comments regarding the vulnerability of the air forces and the resultant implications for the ground combat commander are even more significant. Thus:

...because the Israeli forces placed so much emphasis on the plane, the artillery arm was neglected. Once it is assumed that close support is not available from the Air Force, increased reliance on artillery becomes self-evident...The war taught the incisive lesson that ground forces must be capable of dealing with all problems without depending in any way on the Air Force. Translated into the terms of the field of battle, this requires a very heavy concentration of artillery weapons, so that the Air Force can concentrate on maintaining superiority in the air and intervene in the field of battle in a selective manner.[Ref. 11, p. 271]

If we are to draw any worthwhile conclusions from this most recent modern conflict it behooves us not only to listen to the words of the participants but to examine the implications they have for our own procedures. In addition to examining our own doctrine regarding the employment and allocation of artillery and air power, we are forced to give greater attention to the assumptions made about the general combat uses of these systems. While it is a relatively easy task to call for increased amounts of artillery units and a corresponding change in the mission requirements of the air force (as has recently been done by the IDF subsequent to the October 1973 war[Ref. 9, p. 31]), this adjustment, due





to the nature of the military procurement cycle, would take a minimum of several years to implement, assuming that we actually undertake such a significant reorganization. In the meantime, our decisions for development and future procurement would still be based upon our current models of fire support systems and our general perceptions concerning the nature of future conflict.

During a recent Fire Support Methodology Workshop held at the Naval Postgraduate School, at which such diverse organizations as the U.S. Army's Concepts Analysis Agency and the Combined Arms Combat Developments Activity, the USMC Tactical Systems Support Activity, as well as other government and civilian organizations were represented, one of the major concerns that was expressed by the participants addressed the problem of validating the present series of complex models:

While, as noted above, absolute results of fire support studies cannot be made proof against "reasonable doubt", the models and inputs must be validated whenever possible. This need for continual attention to proof and verification is not unique to military analytical studies. The theories and even the laws of physics are also mathematical models subject to test. However, in the world of physics, the standards of validation are far higher. No theory is accepted for application until it is shown to describe adequately all relevant, previously known phenomena and to predict accurately the results of experiments yet to be performed. In checking theories, great attention is, of course, paid to the precision of experimental results and to the purity and completeness of the data and the model in which they are used. The risks of error, without such rigorous testing, reduce the unvalidated model to the status of so many exotic symbols on paper.

Military analysts pay lip service to these same standards of validation, precision of input, completeness of experiment and model, and to the testing of the sensitivity of results to variations in input. Several workshop participants discussed these requirements on analyses and most have added that these requirements are almost never met. Yet, to an astonishing degree, the discussion then proceeds as though they were! [Ref. 13, p. 32-33]





After recognizing the basic problems associated with validating combat models, the workshop members then go on to call for further improvement within current models as well as some type of testing to be built into the general analysis program to investigate the variability of results due to the basic stochastic nature of the models and their simulated interactions. In an attempt to address this specific recommendation, and to do more than render mere lip service to the problem of complexity and validation, I shall devote the remainder of this paper first, to a brief description of the Field Artillery support system under current U.S. doctrine, followed by a comparison of two models which have a significant impact on the development and employment of artillery systems. My choice of particular models was determined both because of their wide use in the military analysis community and also because they are quite representative of the range of modelling alternatives available to military planners and analysts. Having presented this necessary background material, I shall then propose a specific and relatively simple technique, developed from general queueing theory, which is quite useful in evaluating some of the effects of certain critical assumptions within the current models, such as measuring the amount of variability which is sacrificed by virtue of the choice of distributional assumptions used in modelling the random aspects of some general combat processes. While I have restricted my analysis to Field Artillery systems, due to reasons of personal preference as well as to the increased attention such systems are generating as a result of the recent Mideast war, the overall technique which is presented can be applied to a number of other combat interactions with only slight modification, thus extending the applicability of most of the conclusions of this paper to areas other than that of fire support.



### III. THE FIRE SUPPORT SYSTEM

#### A. MISSION

The basic mission of a field artillery weapons system is defined by the U.S. Army as the requirement:

to provide continuous and timely fire support to the force commander by destroying or neutralizing, in priority, those targets that jeopardize the accomplishment of his mission.[Ref. 36, p. 3]

In order to achieve this overall objective on the battlefield four standard tactical missions have been identified upon which to base combat activities[Ref 33, p. 1-29]:

##### 1. General Support (GS)

An artillery unit assigned a general support mission answers requests for supporting fires from the force artillery headquarters and its own observers. It also has its fires planned by the force artillery headquarters and is positioned by that element.

##### 2. General Support-Reinforcing (GSR)

This mission is essentially the same as the preceeding with the additional requirement on the unit to answer calls from the reinforced element and to establish



communications and liaison with the supported unit. In addition, the reinforced unit may also request that observers be furnished and that, upon approval of the force artillery headquarters, the reinforcing unit be repositioned to facilitate its supporting fires.

### 3. Reinforcing

A unit with this mission remains entirely under the control of the headquarters assigning the mission but has all of its fires planned by the reinforced unit. All other activities are the same as a GSR mission.

### 4. Direct Support (DS)

A unit holding a direct support mission is required to provide close and continuous artillery support to a designated maneuver element and must coordinate its fires with those of the element which it is supporting. It answers calls in priority from the supported unit, its own observers, and the force artillery headquarters. Liaison and communications are established with the supported unit down to battalion level. While remaining under the command of the headquarters assigning the mission (rather than the supported unit), the direct support artillery unit must be prepared to move to other positions as necessary to fulfill its DS mission. The unit also develops its own fire plans. Typically a battalion of field artillery is assigned a direct support mission for a brigade sized force although a single battery or other sized organization can be so employed if the circumstances warrant.

It should be noted in the descriptions of the field artillery missions that the primary distinguishing





characteristic between them is not the manner in which the firing units respond, but rather, the headquarters to which they remain subordinate and the degree of flexibility they have in planning their own fires. Thus, we may infer that the general firing procedures are similar (if not the same) for each type of mission task. Thus I shall examine the direct support mission in close detail and indicate major differences between that and the other missions as necessary. The primary motivating factor for choosing the direct support mission rather than one of the others is that in the standard U.S. division there are four battalions of field artillery of which one is typically assigned as general support for the division as a whole and the remaining three are usually assigned a direct support mission, one to each maneuver brigade. Consequently, the majority of U.S. artillery is employed in the direct support role and any significant combat interactions within this system will have a greater effect on battlefield capabilities and tactical procedures.

## B. THE DIRECT SUPPORT SYSTEM

The direct support system is characterized by three major subsets of tasks which must be accomplished in order for the mission to be successfully prosecuted: targets requiring artillery fire must be acquired by the elements of the system, firing data must be computed, and finally, the projectiles must be fired. If any of these three links is defective, the entire system will fail. If no targets are found, obviously there is nothing to shoot at and thus the other elements of the system are not even required to function; if the firing data is not computed, or computed incorrectly, the required effects against the target are not achieved; finally, if the projectiles are not fired there is



no effect on the battlefield even though all the other elements of the system may have functioned perfectly. In addition, since artillery weapons are usually employed in an indirect fire role (i.e. no line of sight exists between target and artillery weapon) successful communication between each of the three major components is a necessity. For the sake of reference I will classify all functions of target acquisition as belonging to the Forward Observer (FO) element; all computational duties as the province of the Fire Direction Center (FDC) element; and the firing tasks as belonging to the Firing Battery (FB) element.

#### 1. Functions of the Forward Observer (FO) Element

The primary duty of the Forward Observer element is to provide target information to the direct support system and to engage confirmed and suspected enemy locations in support of the overall tactical plan. As the artillery representative at the company level, the forward observer functions as a coordinator of supporting fires and is an advisor to the maneuver company commander. During movement, the FO element travels with the supported unit and takes under fire targets of opportunity as well as determining when to request the firing of preplanned targets. In relatively static situations, the FO element assists in planning defensive fires as well as advising the maneuver company commander on the proper employment of indirect fire weapons.

In a typical conventional war scenario there are usually three forward observer teams assigned per firing battery, making a total of nine elements in a direct support battalion. To assist the maneuver units in overall coordination of the fires that will be requested and planned by the various FO elements, there is a liaison officer (LNO)



assigned to work with each supported maneuver battalion. The primary duties of the LNO include not only the coordination of fire requests from the FO elements, but also the elimination of target duplication, as can occur when two separate FO teams have the same enemy position in view and desire to take it under fire. The LNO then acts as a type of filtering agent to insure that only the necessary missions are fired, usually by specifying only one of the observers to adjust fires on the target. In addition, the LNO assists the battalion staff in planning preparation fires and other types of preplanned concentrations to be called for during various phases of the combat operation. In this respect, the LNO is an integral part of the Forward Observer element in that he also serves as an originator of targets to be fired by the direct support system.

In the other mission categories many of the duties of the Forward Observer element (such as determining the location of targets to be fired) are performed as a staff function rather than by personnel in the field with the maneuver units. For example, GS and GSR units depend to a much greater degree on target input from various sensor systems (seismic, magnetic, etc.) as well as aerial photography and intelligence data. However, in that the entire purpose of these other support operations is to produce targets to be fired, we can say that the primary difference between these units and those with a direct support mission is characterized by the source of target data rather than any other factor. Thus, while not technically a "forward observer" in the line unit sense, the higher staffs perform similar target acquisition duties which lead to the activation of the fire support network.

## 2. Functions of the Fire Direction Center (FDC) Element





In a direct support unit the Fire Direction Center element is comprised of two stages, the battalion and battery Fire Direction Centers. The battalion FDC serves as the battalion coordinating center to insure that supporting fires are initiated promptly regardless of the tactical situation and it is here that the allocation of missions to specific firing batteries are determined based on such factors as whether the battery is already engaged in a different mission, the amount of ammunition of the required type available in the several batteries, etc. Although it is standard practice for a particular battery to support the same maneuver battalion or task force on a regular basis, this is not always possible since the battery may be moving to a new position or may already be engaged in a higher priority mission. Thus, the battalion FDC serves as the allocation center by which the next available firing battery is chosen to fire the requested mission. Also, in the case of several missions arriving in a close time frame, the battalion operations officer (S-3) or Fire Direction Officer (FDO) usually decides on the priority to be given to the arriving missions in the event that there are more requests for fire than there are batteries available, even to the point of aborting an on-going mission if such need should arise. In addition, the battalion FDC also attempts to provide a data check on the computations of the initial rounds of a mission in order to preclude any possibility of error.

The individual battery FDCs, after an initial verification of the firing data, usually control the entire mission and maintain direct communication with the FO element until notified that the mission has been completed either because the required effects have been achieved or the target has moved out of the sector of fire. In a mission against a target of opportunity it is often necessary for the FDC to receive adjustment information from





the FO in order to achieve and maintain a high volume of effective fire on the target which may be moving away or taking evasive measures. In the case of unobserved fires, the battery usually fires the requested number of rounds and then asks for further instructions, while in the case of preplanned fires, such as defensive targets and final protective fires, the necessary firing data is kept readily available on call and regularly updated for changes in weather, powder temperature, etc.

The firing computations in the FDC at both battalion and battery level are performed on the FADAC (the M18 Gun-Directional Computer) and are supported by a manual back-up system. Thus, it is possible for a battery to be engaged in several missions simultaneously since the computer can handle two different missions and the manual back-up can also be used on a separate mission if necessary. However, due to the limited radio nets available, this poses a severe communications problem unless there is an augmentation of available radios and assigned frequencies for the artillery system or unless one or more of the requestors has a land-line telephone link with the firing unit. In either case, while more than one mission may be in progress at a particular firing battery at the same instant, the number of weapons per target is reduced to only a fraction of the battery per mission. Thus, in order to achieve a given level of target effects with a reduced battery, more separate volleys must be fired and consequently, a greater mission time is accumulated by the reduced battery.

### 3. Functions of the Firing Battery (FB) Element

Once the target has been acquired and the appropriate firing data computed, the firing battery element



enters the operations picture. At this level the weapons are maintained, the ammunition fuze and readied for firing, and, when in receipt of a mission, the required data is set on the weapons and the projectiles are fired. The firing battery communicates only with the FDC and thus is primarily a reactive agent rather than a causative one in the operation of the fire support system. However, errors at this level are still quite serious since they may cause the FO element to make erroneous adjustments and thus the required target effects may not be achieved.

In a typical direct support firing battery there are usually six howitzers. The size of the weapons depends primarily on the force being supported and the movement capabilities required to maintain adequate support. Thus, an armored or mechanized infantry division will generally be organized with self-propelled artillery units (usually 155mm) and airborne and air mobile forces will be supported by the lighter, towed 105mm weapons. In addition to the mobility criterion is also one of range limitation. Usually the larger weapons will also have a longer range capability. Thus, in the case of supporting a mobile ground force, the longer range 155mm weapons will not be compelled to interrupt their supporting fires in order to maintain proximity to the supported units on the battlefield as often as would the shorter range 105mm weapons. However, this does not preclude the necessity of remaining mobile for the purpose of avoiding enemy counterbattery fires, which is an inherent task for all firing battery elements.

#### 4. The Completed System

Tying together the major pieces of the direct support system as described above, we thus see that the chain of events required in a fire mission are linked



together as in Fig 1. A mission originates with the FO element as either a target of opportunity or one of several types of preplanned missions. Unless there is an emergency, the liaison officer acts as a filter through which the fire requests are transmitted and he eliminates any possible duplication in both the planning and firing phases of the operation. Once the final target lists are generated, they are transmitted to the battalion FDC for allocation to the appropriate firing batteries and, in conjunction with the individual battery FDCs, the necessary ballistic information is computed to insure that the rounds are delivered on the targets. When fire is finally called for, the battery FDC sends the appropriate data to the firing battery and the rounds are fired. Should subsequent adjustments or additional rounds be required, the FO element responsible for the mission contacts the battery FDC directly with the latest change in the mission requirements. Thus, this description characterizes the Direct Support System and, with the exceptions or differences as noted in the above paragraphs, also provides an adequate description of the other artillery fire support systems as well. With this information now having been presented, I shall proceed to examine how this system is treated in current models as used by the military analysis community.





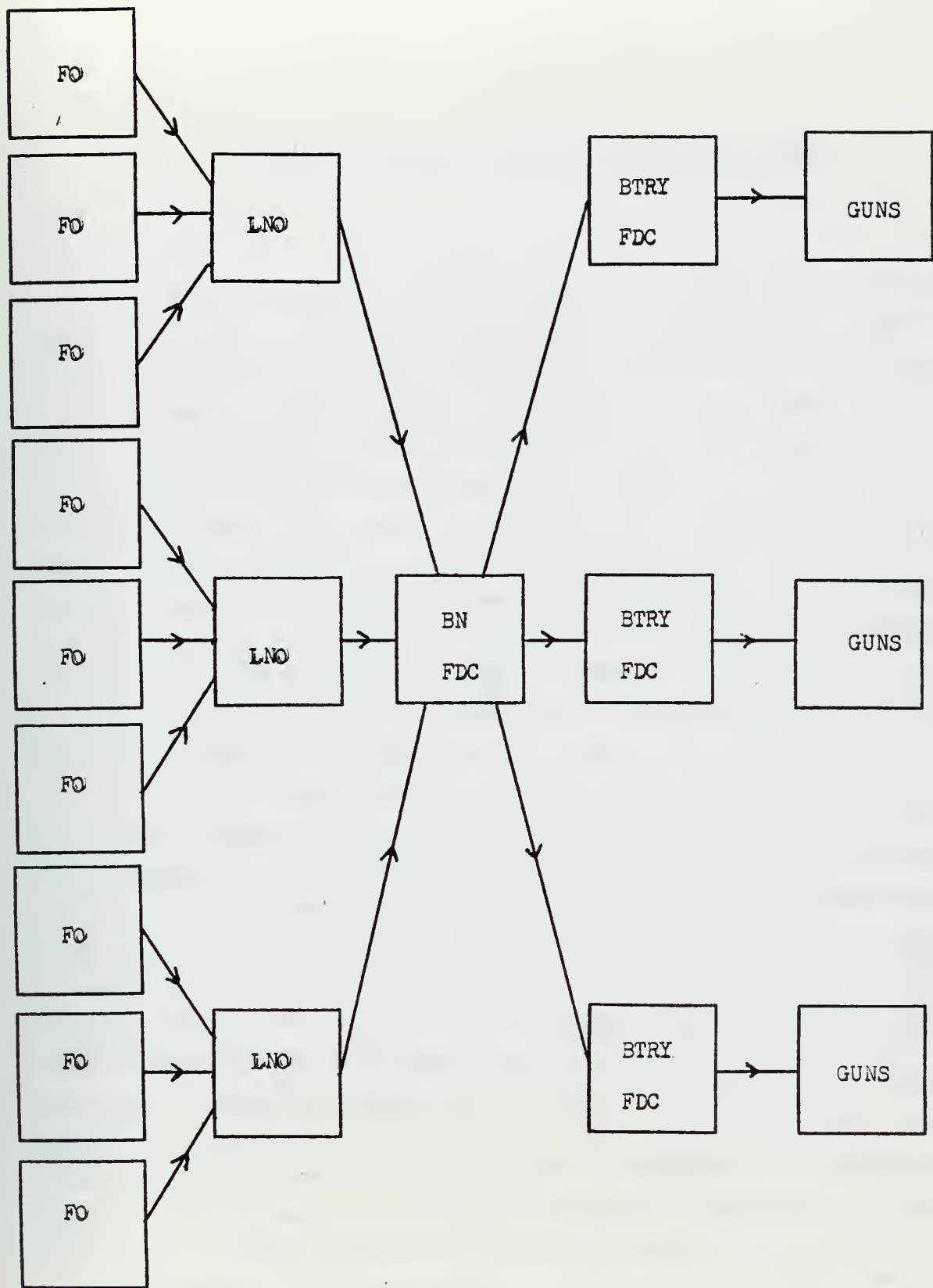


Figure 1 - THE DIRECT SUPPORT SYSTEM





#### IV. CURRENT MODELS OF THE ARTILLERY SYSTEM

Prior to examining some of the more widely used combat models and their treatment of artillery problems, it should be noted that there are two different philosophies regarding the purposes of combat models in general. The first holds that the purpose of a model should be to provide inferential information to the combat commander. That is, models of warfare should be used primarily to identify trends in combat activities and resultant techniques or courses of action which the combat commander may choose to employ on the battlefield. In direct opposition to this is the second theory, which holds that models should be so detailed as to be predictive in nature. Thus, with the appropriate inputs and descriptors of the combat process, the model should be able to foretell the effects of proposed weapons systems or doctrinal changes with a high degree of accuracy. While the first school of thought had its origins in the development of Operations Research during World War II, it has generally been neglected during the present era with the possible exception of those individuals who work entirely with the highly aggregated models of Lanchester origin. Meanwhile, the second school of thought has come into prominence in the military analysis community, largely assisted by the increased emphasis on planning and budgeting that was introduced into the US Department of Defense by Secretary Robert S. McNamara, with the concomitant result that almost all major studies performed in the US today are intended to be predictive in nature.[Ref. 3] With this in mind, we may now proceed to examine some of the models currently in wide use in the military analysis community.



Given the wide range of military problems of current interest, it is immediately apparent that there are a multiplicity of models to examine which range from very detailed, high resolution simulations such as DYN-TACS and CARMONETTE, to large scale, highly aggregated theater models such as the Legal Mix series and the Non-Nuclear Ammunition Combat Rates model used by the US Army Concepts Analysis Agency (CAA). Thus, in order not to be overwhelmed by the task of describing each of these models in detail (as well as the numerous others not specifically mentioned), I shall limit this section to an examination of one model from the high and low resolution ends of the spectrum to illustrate current treatments of artillery problems. I have accordingly chosen the DYN-TACS model because of its extremely well-done documentation and the CAA AMMO RATES study, not only because of my personal experience with it, but also because this particular combat model is presently (April 1976) being implemented as a major predictive study tool for NATO forces in Europe.

#### A. THE NON-NUCLEAR AMMUNITION COMBAT RATES (AMMO RATES) MODEL

The purpose of the AMMO RATES methodology [Ref. 29, 30] is to provide measures for all combat ammunition, from small arms and hand grenades all the way up to large caliber artillery rounds and Field Artillery missiles. Thus, the overall model is composed of various submodels which examine in detail specific aspects of the combat process. These submodels include the Infantry Combat Model, the Tank Antitank Model, two helicopter models which are used in both an antiarmor and antipersonnel mode, a Target Acquisition Model, an Artillery Casualty Assessment Model, the Red and Blue Artillery Models, and other associated routines for



assessing the effects of tactical air support and air defense artillery throughout the theater. Since we are here concerned primarily with artillery systems, I shall elaborate further on the Blue Artillery Model and only discuss the other submodels as may be required to facilitate understanding the interactions of interest in the Blue Artillery Model.

The Blue Artillery Model (BAM) is designed to simulate the allocation of artillery fires on a 100 by 100 kilometer section of the battlefield for artillery units down to battery and section level. In order to perform this task, the location of all friendly artillery units are provided as inputs to the model along with tables indicating which available ammunition types are effective against the 16 categories of possible enemy targets that are playable and the attack and defeat criteria for each target category and size. In order to provide enemy targets to be fired upon, the Target Acquisition Model (TAM) is run to determine which enemy elements within the battle sector have been discovered. The TAM assesses the size, type, and location of the enemy units and also assigns acquisition times to each target thus identified. The final output thus consists of a list of detected targets which includes the target identification number, the time of acquisition, coordinate location, category, environment (woods, open or town), mobility, estimated and true size, type of sensor that detected the target, and the number of enemy personnel and tanks which are estimated to be vulnerable to indirect fire at the time of acquisition.

The BAM takes this target list as an input to its event store simulation. During the running of the BAM, the acquired target list is consulted and targets are processed on a first in, first out basis. As each target comes up for consideration, the BAM evaluates each battery in the firing





force to determine which ones can engage within three minutes (the assumed minimum processing time to fire a mission). Having determined which units are thus available to react, the model determines which of these have the required types of ammunition available to inflict damage against that particular category of target in the target's sensed environment. In the event that more than one type of munition can achieve the desired effects, the model chooses that weapon-munitions system which will provide the required damage at the least cost. If no ammunition available to the model can achieve the required damage level for defeat, then the particular weapon-munition combination which achieves the highest damage level of those available to the model is allocated to fire on the target. Once allocated to a given battery or group of firing units, the fire mission time for that particular target is calculated in order to determine when the engaged artillery units will next be available for assignment to another mission. In the process of these calculations, the model uses the number of rounds that must be fired per weapon and divides this by the weapon's sustained rate of fire (which is an input to the simulation) to arrive at the total mission time.

It is readily apparent from the above discussion that the BAM is primarily a deterministic model with a limited random input. All of the decision rules for allocating a mission to specific firing elements are specified, as well as the rules for determining the time length of the mission. The major random aspect of the model is the input of the acquired target list as the output of the TAM. Thus, as long as the same target list is loaded into the model, regardless of the random number seeds used for some of the casualty assessment routines, the allocation of batteries to fire, and their subsequent required firing times, will be the same from one run to the next. This allocation will only differ if the targets on the TAM input are shuffled





into a different order or are replaced by different targets. As a result of this technique, the model appears to have one significant area of danger which deserves some mention. That is, the BAM does not account for possible requirements to adjust artillery fire against moving targets on a round-to-round basis. If a moving target is acquired and can be fired upon within the time limits specified by the decision rules of the simulation, no attempt is made to vary the rate of fire to account for the target's motion, nor is the ballistic aim point changed. Thus, from an examination of the model input requirements, there is no special information required to adjust the weapon effects for a moving target. Consequently, the model is firing rounds at one particular aim point when the target may have already left the area after the first rounds landed. This results in either an over estimation of the casualties inflicted (because the target is no longer at its reported location against which fire is being directed) or an over estimation of the number of rounds to be fired since, if this mission were under the control of a forward observer, as soon as the target departed the firing zone the mission would have been halted or the following rounds adjusted onto a new location. This has additional effects on the model because, while units are busy firing these unnecessary rounds which may be having no effect (unless the target was crippled on the first volley), other worthwhile targets may be lost because of the erroneous weapon allocation. However, it must be kept in mind that this is a theater level model and thus must not attempt to simulate all possible combat activities, as that would be prohibitive in terms of both time and budget resources. Rather, we are here raising problems which must be handled at a different level of resolution and therefore let us now turn to a glance at the high resolution DYN-TACS model.



## B. DYNAMIC TACTICAL SIMULATOR (DYNTACS)

The DYNTACS model was first operated in March 1967 after having been developed at Ohio State University under a government contract. As originally designed, the model was intended as a small unit level, high resolution simulation. In contrast to the theater model discussed earlier, DYNTACS is capable of representing a battlefield sector of an approximate size of 5 by 10 kilometers, with some variation in these limits due to core storage capacity available on different computer systems. Because of the emphasis on high fidelity representation of small unit activities (down to the size of crew served weapons), much of the model is concerned with terrain and mobility problems and the ability of the small tactical units employed to detect enemy targets. Unlike the BAM, DYNTACS is a two-sided simulation in that both friendly and enemy forces react to one another's moves and fires whereas the BAM is one-sided only. In addition to the detailed mobility and firing routines in the model, a great portion of the run time is devoted to the computation of line of sight and other detection related problems, including modeling the various communications networks over which information is passed by the various combat elements.

The artillery module of this complex program is composed of three submodels: the Forward Observer, the Fire Direction Center, and the Firing Battery models. Each of these routines are further broken down into events as follows:

### 1. Forward Observer Events

#### a. Self-Defense



Since the forward observer element is represented as accompanying one of the mobile combat elements in the model, this routine simulates the basic measures taken by the FO party if endangered by enemy forces. For example, if the FO is mounted with a tank unit, the model assumes that when the tank is engaged in a direct fire mission which requires the entire crew's attention, that the FO will not be able to adjust fire indirectly. Similar types of restrictions are imposed when the FO element is on foot or mounted in an APC (armored personnel carrier).

#### b. Selection of the Target

The Forward Observer first checks to determine whether any preplanned, on-call fires are required by the combat activity. If the result is negative, the selection routine then evaluates all detected enemy targets using a series of weights which are input to the model in an attempt to assign relative values to the various targets based on responses to such questions as: what is the enemy element (tank, APC, or crew-served weapon); is the target location known or only suspected; is the element moving or firing; is the target already receiving friendly fire; etc. After considering all known targets, the one with the highest weighted score is chosen for attack and a fire request is prepared for communication to the Fire Direction Center.

#### c. Communication with the Fire Direction Center

This event requires that the FO check to determine if his radio net is open and, if so, the message is sent to the FDC. Since it is assumed that several fire





missions may be handled by the FDC simultaneously, up to a specified maximum which is user input, if the FDC can handle the incoming mission the FO prepares to observe and adjust fire. In the event that the FDC is saturated, the FO is told that the mission cannot be fired and he returns to the target selection mode.

#### d. Adjustment Procedures

Once a fire mission is accepted, the FO is placed in an adjustment mode and, after each volley, must determine whether to enter fire for effect, continue adjusting, or terminate the mission. Depending on the model's criteria for damage levels, the FO selects one of these alternatives and attempts to communicate his response to the FDC. This communication procedure is essentially the same as that described for initiating a fire mission with the exception that the messages will usually be of shorter duration.

## 2. The Fire Direction Center (FDC)

This particular submodel is much simpler than the FO section due to factors noted previously, namely, the high degree of emphasis placed on detecting enemy targets and moving over the battlefield. For the purposes of the DYN TACS model, the FDC and the firing battery elements are not required to be mobile since they are usually located and employed behind the FEBA rather than as an integral part of the forces on it. Consequently, the FDC is primarily a transmitting and receiving center where fire missions are received, data is computed for firing, and the resulting information is sent to the firing battery over telephone lines. Thus, once the FO element makes contact through the





communications network, the fire mission is assumed as having arrived at the FDC.

When the FDC portion of the model becomes the current event, after an interval of time representing the transmission of the fire request, the routine checks to see if the FDC can handle this new mission. Current doctrine specifies that two missions can be computed simultaneously given the current organization and personnel assigned at battery level. However, the DYN TACS model is capable of being adjusted so that more missions may be fired, up to a user specified maximum. In the event that the FDC personnel are fully utilized when a new mission arrives the model assumes that the new mission will remain in a standby status in a queue with the radio operator until one of the computing elements is freed from its current task. If a second overload mission should arrive while the first is still waiting, the model informs the FO element that this newest mission cannot be processed and the particular FO sending the mission reverts to the search mode.

The computations of the FDC are represented by drawing from a random number distribution to determine the amount of time that elapses until the firing data is computed and sent to the firing battery. When this action is completed, the FDC reverts to a standby mode until another message is received from either the adjusting FO or until a new fire mission arrives.

### 3. The Firing Battery Events

This routine performs three functions: it simulates the duties of the firing battery personnel in fuzing and firing the munitions, it determines the firing assignments of the battery, and it conducts the damage assessment



functions for the artillery portion of the model.

a. Battery Duties

The duties of the firing battery personnel are simulated by drawing from a random number distribution to reflect the length of time that would be required to prepare the weapons for firing in the adjustment and fire for effect phases. This is a simple function and is similar to the drawing in the FDC event to determine the time for calculations.

b. Firing Assignments

Although this is usually done in the FDC in actual practice, the PYNTACS model has altered the order in an attempt to simplify the running of the program. That is, rather than require the firing battery to communicate with the FDC a second time merely to indicate that the rounds have been fired, this activity is handled at the firing battery level. This change can be justified by observing that the transmission of this particular message is of extremely short duration and is thus not significant in view of the fact that the firing battery has an immediate communications link with the FDC by phone line and is thus not subject to a waiting phenomenon as is the FO when he attempts to enter the radio net to send his messages.

When the firing battery element completes the last volley of a mission, the model checks to see if any other missions are waiting to be fired. If there are none, then the battery sets its next event flag to the time of the next set of scheduled fires and becomes idle. On the other hand, if there is a mission pending, the model estimates the



time required for firing the mission based on the number of volleys required and the average firing time per volley. If this estimated mission duration does not interfere with any scheduled fires, the firing battery accepts the task and begins firing.

If the estimated completion time conflicts with the next set of scheduled fires, the model checks to see if the requested mission can wait until after the scheduled fires have been completed. If this is not possible, the routine then determines if the scheduled fires can be delayed up to a user specified maximum length of time. If the conflict is still not resolved by means of any of these steps, the model then determines the weights of both the scheduled and the requested mission and the higher weighted mission is fired. In addition, should the scheduled mission have the higher priority, the firing battery element checks to see if at least one volley of non-adjusted, fire for effect can be placed on the requested target prior to initiating the scheduled fires.

Once a particular mission has been accepted for firing, the firing battery element reverts to a standby status after each volley of an adjustment until it receives further instructions. upon receipt of those instructions, the routine again checks for any possible conflicts with scheduled fires for the remaining volleys as described above and executes the mission now having the higher priority. However, once the firing battery element enters the fire for effect phase, all of the requested volleys are completed prior to assigning any other missions.

### c. Casualty Assessment

During the firing, a damage subroutine is called





after each volley in order to determine the effects of the latest rounds fired. This routine computes the ballistic aim point for each weapon being fired based upon the target coordinates and the particular artillery formation being employed by the firing elements. Since the artillery model assumes that no-fire-lines are placed around friendly elements with sufficient safety factors as to preclude producing casualties among our own troops, the casualty routine only scans the list of enemy forces to determine which are neutralized or damaged.

### C. AN OBSERVATION

While this brief description of two particular models was not meant to be all inclusive, nevertheless, the techniques of simulation which are employed in the two examples chosen are typical of most combat simulation models and as such are widely used in military analysis. That certain of the current models have also gained some measure of partisan support, especially from their designers and proponent agencies, as they have become more widely used, cannot be ignored. However, no one modeling technique is appropriate for analyzing all military problems, nor are all current models fully capable of being adapted to study many of the problems for which they were not originally designed. In many cases, such a modification of calculations or output would be more expensive, in terms of time and effort, than in building a simpler, smaller model to address a specific question. For that reason I propose to examine a different technique for modeling artillery allocation and employment than is currently available in most widely run models.



## V. A QUEUEING APPROACH

In the last chapter two representative combat models were examined in some detail in order to provide some insight into the manner in which Field Artillery problems are currently handled within the military analysis community. A simple comparison of those two models reveals that not only is there a significant difference in scale, i.e. DYN TACS plays battalion and smaller unit actions versus the theater level activity of the AMMO RATES model, but that each plays the random events of combat and human activity with a differing degree of resolution. Thus, DYN TACS utilizes numerous random number routines to describe combat interactions whereas the CAA model is essentially deterministic once the target arrival times have been determined. While these differences of modeling approach are the ones most often subject to discussion as to which is the most appropriate for the particular problem under consideration, the primary purpose of the entire family of military models is nonetheless the same - to predict the effects of current and proposed weapons and variations of force size in different combat situations. This is most easily seen in the case of the AMMO RATES methodology, where the entire purpose of the study is to provide rates of ammunition expenditure for various intensities of combat in both the European and Asian theaters, which then have a direct influence on the requests of the U.S. Army for the production and stockpiling of appropriate quantities of munitions. The predictive nature of DYN TACS and other military models, on the other hand, is usually a bit more obscured because these other models are not typically a part of a regular series or compact methodology but are, instead,



employed in Cost and Operational Effectiveness Analyses (COEAs) depending upon which of them, either singly or in combination, seem more appropriate to the problem under consideration. Thus DYN-TACS has been used in the XM1 study, the Hellfire COEA, and the Cannon Launched Guided Projectile (CLGP) COEA (concerning which I shall say more later in this paper). However, it is this predictive purpose behind military models which has had a profound impact on the development of both the military modeling community and the models themselves in two important aspects which are intimately interconnected.

The first of these effects (which are rarely appreciated in their full significance) is that of the requirement for realism. Due to the implicit predictive uses of the models, it becomes vitally important that the models chosen for a particular study are not only appropriate in that they are concerned with the combat activity of the weapons or system under analysis, but also that they represent real life with as great a fidelity as is possible within a computer model. While no worthwhile analysis agency will pretend that this problem of the realistic portrayal of combat activity has been totally solved by any one model, or any particular combination of models, nevertheless, the striving for realism is of the utmost importance if billion dollar contracts are to hinge upon the results of the study. However, we now find ourselves in a serious predicament for, when confronted with the body of combat models in existence, how do we choose the "most realistic" for the purposes of our study? The answer to this question leads us directly to the second effect referred to above - namely, that of complexity in the models.

Since there is a dearth of "hard" data regarding combat interactions, and the dimensions of this vacuum expand considerably if we restrict ourselves to combat data and





exclude the results of field experiments conducted under simulated combat conditions, we find ourselves forced to look at some other criterion besides the nature of the inputs in order to determine the degree of realism of a model. Under these circumstances, the military analysis community has generally opted for the more complex model over the simpler one. Thus, if one model has subroutines to simulate tank-to-tank radio messages and an alternative does not, the more complex model is generally considered to be the more realistic, even though no factual information regarding how radio messages influence the course of a battle may be available with which to measure the "realism" of the communications subroutines themselves, let alone estimate their effects on the remainder of the model's interactions.

Thus, in order to justify the models as predictors of performance, or, at a minimum, as indicators of general trends which can be associated with varying weapons systems or force alternatives, the modeling community has been compelled to build more and more complicated models. It is at this point that the objection may be raised that not all models are as complex as DYN-TACS and that, in fact, a great deal of aggregation of combat activities takes place, even in battalion level simulations, especially in models based on the Lanchester theory of combat. However, we must not allow this argument to gain undue influence, for it is often overlooked that the data supporting the attrition rates and other factors required by a Lanchester-type model are not always gathered by field experimentation or as the direct results of actual combat, but instead are generated by the higher resolution models and then used as inputs to the more aggregated simulations. This incestuous relationship has some great dangers associated with it. While the more highly aggregated models may be theoretically valid indicators of combat trends and general weapon performance,





the accuracy of their predictions depends directly upon the accuracy and reliability of the output from the high resolution models.

Recognizing, then, the critical role which the high resolution models play in the analysis of military problems, we must ask the question: how have these simulations been verified, or at the very least, what work has been done to evaluate the sensitivity of the models to the assumptions made regarding not only the ranges of possible inputs but also with respect to the nature of the random number distributions used in generating the combat interactions? It is at this point that the interrelated problems of realism and complexity exert their influence, for in a model like DYN TACS not only are there a multitude of factors to be examined, but to produce a full design matrix which covers just those alternatives which are feasible as well as reasonable in military terms, would be quite prohibitive in terms not only of money and man-power requirements, but especially in regards to computer time. For example, a typical run of the DYN TACS model can take on the order of 60 to 90 minutes to represent a battle which, in actuality, would be completed in 30 minutes in the tactical environment. Thus, to get estimates of the model's response due to varying just one of the many parameters of interest would require several hours, or even days, of computer time since no statistician would like to stake his entire analysis upon a sample size composed of one run. Thus, for each alternative that we might wish to examine, we would require a minimum of several hours of model time in order to get reliable data and to do this for all possible alternatives of interest is most definitely beyond the range of reasonable expectation.

Having illustrated the difficulties associated with verifying or analyzing the sensitivity of the high



resolution models, we are now in a better position to answer the question raised previously, i.e. what has been done in this area? Unfortunately, the answer turns out to be: little, or nothing at all. This shocking reply is substantiated by the following extracts of information from the Tank Weapon System Final Report and the CLGP COEA:

Since data were not available to determine appropriate distributions to describe the various artillery response time parameters, the artillery models, as an interim measure, assume that such variables are described by normal distributions whose parameters are specified as input...Although there is no data to validate the assumption in this model, the normal distribution has been found to describe a large number of physical processes. The models further assume that the variability of the response time distributions is sufficiently small such that the probability of obtaining negative Monte Carlo samples from the distribution is negligible, i.e. if "z" is distributed  $N(\mu, \sigma^2)$ , it is assumed that  $\mu > 4\sigma$ , where  $\mu$  and  $\sigma$  are the mean and variance, respectively, of the normal distribution.[Ref. 2, Chap. 10]

While this citation from the DYN TACS documentation was certainly nothing to complain about in 1969, when the final report was published and the model had not yet become widely used as an analysis tool, it appears that nothing significant has been done since that time to verify those assumptions mentioned above. This fact is demonstrated in the CLGP COEA which used the DYN TACS model for part of the study and the assumption was made that the variables of the artillery response time distributions were such that  $\sigma = 0.2\mu$  for the purposes of the study.[Ref. 31, Vol. 6, Chap. 4] In addition, Fig 2 presents the table of the assumptions made in the CLGP COEA's analysis of the impact of the CLGP on present communications procedures within the field artillery.[Ref. 31, Fig. M-25] It is significant to note that the most cited reason for the assumptions is "lack of standard criteria", even for interactions which are not due solely to the introduction of the experimental CLGP system, and also that the validation process seldom mentions field



ASSUMPTIONS BY MAJOR CATEGORY	REASONS FOR ASSUMPTIONS	VALIDATION PROCESS
1. THREAT a. Symmetrical attack b. Pcisson arrivals c. Speed of attack equal to 3 meters/second d. No target masking e. Targets of opportunity occur in clusters with a mean of 3.3 targets per cluster f. Targets served on a first-come first-served basis	Lack of standard criteria and limited scope of study	Field Artillery School approval
2. FORWARD OBSERVER a. Can handle 2 missions simultaneously b. Lase only for the first round c. 10-20 seconds to make mission assessment d. Mission time per target measured from detection to end of mission message (FO does not request additional fires)	Lack of standard criteria	Field Artillery School reports and School approval
3. COMMUNICATIONS a. Error free environment b. Lase message has priority c. Rounds complete message is not critical d. Missions generated only during normal pauses e. Message time 2.5 seconds to start transmission plus 0.35 seconds per word	Lack of standard criteria	Field Artillery School reports and School approval
4. BATTALION FDC a. Net control station b. Can handle 2 missions simultaneously c. Computation time is 2/3 trajectory time	Lack of standard criteria and limited scope of study	Field Artillery School approval
5. FIRING BATTERY a. Guns ready 40 seconds after receiving data b. 0.6 seconds to recycle for a new mission c. Unloading takes only 15 seconds d. Rounds fired not transmitted if round has impacted prior to message e. 20 seconds between successive volleys f. Trajectory time (seconds) equals $4 \times \text{Range}$ (in thousands of meters)	Lack of standard criteria and limited scope of study	Field Artillery School approval, reports and test observations

FIGURE 2 - CLGP COEA COMMUNICATIONS STUDY ASSUMPTIONS





experiments or actual test results. Indeed, this is a perfect example of being unable to see the forest for the trees and, as the modeling community is currently developing, more and more time is being spent on examining the leaves rather than on some of the overall effects due to forestation in general.

It is precisely at this point, however, that the use of queueing theory to examine the general results of complex interactions can make a significant contribution to an understanding of the effects of different distributional assumptions in the modeling of field artillery problems and in determining which areas would most likely provide fruitful results after further experimentation and investigation. The use of a queueing approach is also eminently satisfactory from the view point of handling the problem of complexity for it will allow us to eliminate many of the extraneous details which lend little direct insight into the general problem but which greatly complicate the predictive models and thus tend to obscure significant interactions and relationships. To illustrate how queueing theory may be advantageous for our purposes I shall set forth the following situation as an example.

#### A. A USE OF QUEUEING THEORY

Let us suppose that we are tasked to analyze the operation of a small airport facility in order to determine whether present handling procedures could be changed in order to provide better service. In this scenario we will assume that the airport has two landing strips and provides ground services for three different airlines which are capable of handling only one aircraft at a time at their respective terminals, as well as some general services for



some locally based aircraft which are privately owned. In addition, we will assume that the geographical layout of the airport is such that once any particular aircraft has completed ground servicing it must wait at the terminal until a take-off runway is available, thus blocking any other planes from service. We will further assume that each of the airlines uses our facility only as an intermediate stop and thus does not leave aircraft on the ground for extended periods unless due to mechanical failures. Since our primary considerations are to improve overall service, it is obvious that the major beneficiaries will be the visiting aircraft since they have time schedules to keep and thus suffer greater inconvenience if there are significant delays in the service activity.

As we examine this situation, we initially observe that there are certain distinct phases to a servicing operation at the field in question. First, the incoming aircraft must be handled by the control tower and directed to one of the available runways to land. Following the landing, the aircraft taxis to the appropriate terminal entrance and begins its ground service which, in this case we will assume, consists of a cargo and passenger unloading phase, a refueling period, and a reboarding phase. After these actions have been completed, the aircraft contacts the tower and is assigned a runway for take-off and proceeds on its journey. At this point all service activity for that particular craft have been completed.

Since we assumed that only one aircraft can be serviced at a time at either of the three airline terminals, there appear to be two places in which waiting lines (queues) can form; i.e., in the air while waiting for a landing-strip to become available, and on the ground while waiting for access to a terminal facility. Obviously, the degree of congestion that will be apparent is heavily dependent on the aircraft



time schedules which determine the general arrival rate of planes (customers) to our airport, and also on the amount of time necessary to complete the various ground services. As our nearby metropolis begins to grow, the airlines servicing it will come under increasing pressure to increase their flights to this area and thus the need for our study becomes readily apparent. It is also apparent at this point that we can make improvements at our airport in various ways. We could simply enlarge the ground waiting area and increase the number of runways, thereby reducing the queue in the air, along with the dangers associated with keeping aircraft circling overhead; we could enlarge the terminal facilities to handle more than one aircraft at a time; we could improve on the service times at the various stages in the ground process; or, finally, we could initiate some combination of all of these methods to improve overall service.

Our queueing formulation is now admirably suited to determine which of these alternatives will yield the greatest benefit, for we can model the entire system along the lines of Fig 3. Here, we see that our arrival stream (aircraft requiring service) passes through several stages composed of the control tower (first server) and then onto a particular terminal facility (second server) depending on the airline involved. At the second server, the various stages of ground operations are completed in sequence and the aircraft is then returned to the tower (which now functions as a third server) for final clearance and take-off. With this formulation we have completely specified a queueing system and it only remains to gather some information about the distribution of service times at the successive stages in order to arrive at a complete specification of our own particular facility. Unfortunately, there are only a very few distributions which we might apply to our service stages which would yield solutions which are analytically tractable. Most





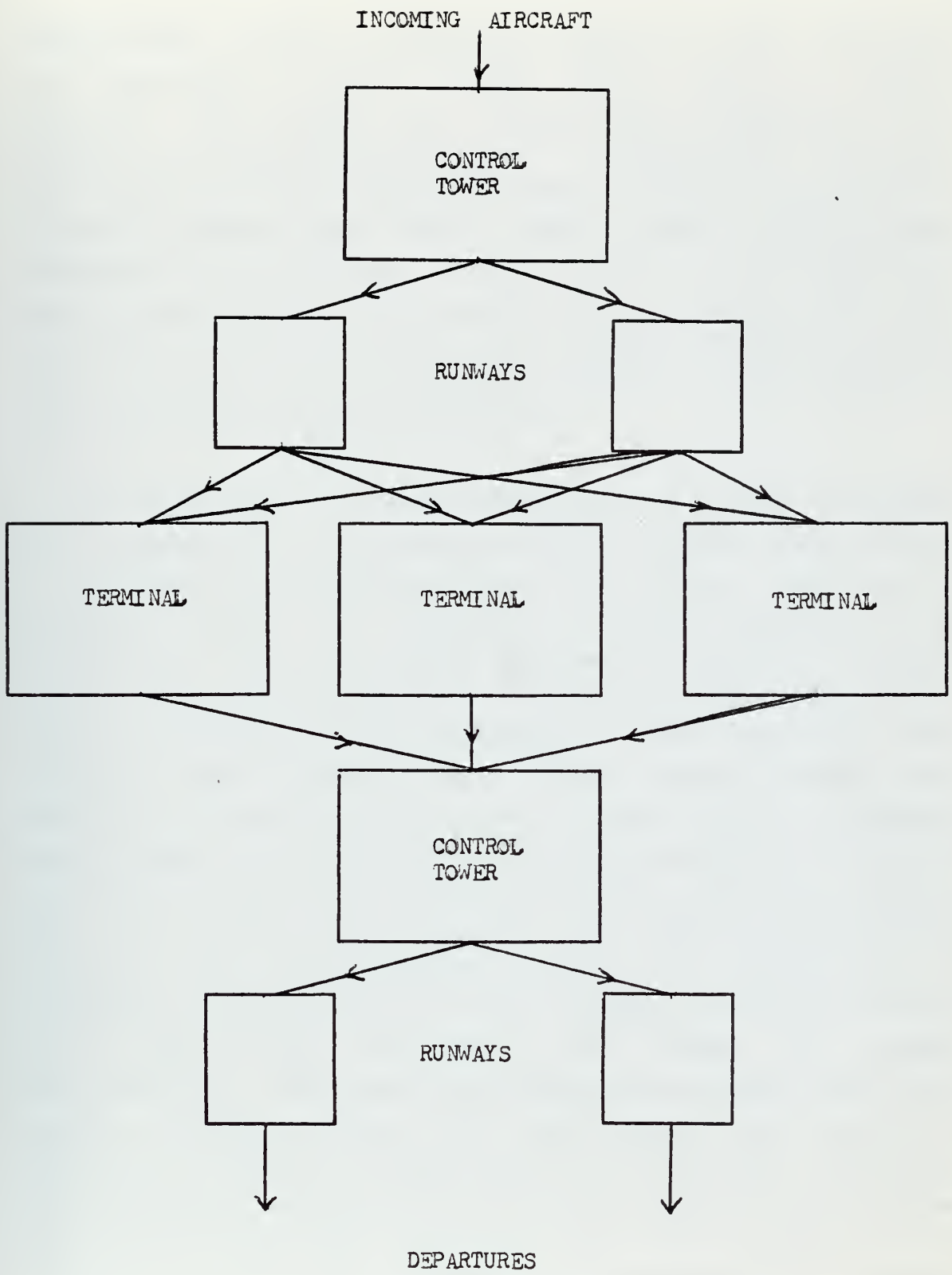


Figure 3 - THE AIRPORT EXAMPLE





combinations usually result in a system of state equations too complicated to solve without using either a digital computer for solving the differential equations which result, or in building some sort of simulation. However, even for the most general type of composite service and arrival distributions, there exist useful computational formulae for calculating such factors as the probability that a delay occurs; the expected total service time per aircraft; the expected number of customers (aircraft) in the queue; and the server utilization factor, to name some of the most common items of interest.

At this point it may be argued that if we have to resort to a simulation to gain results for our system, what use was it to formulate the problem so as to require the use of queueing theory? In response to such a question, it is sufficient to note the immense amount of extraneous detail which we were not required to use in order to formulate our problem or to put into a simulation of our facility. For example, when examining the ground service stage which involves the refueling procedure, nowhere did we require subroutines or equations of sub-stage activity to specify the manner in which the fuel truck drives out to the plane, how the nozzle is connected, etc. This is particularly important in three respects. First, by being able to amalgamate these actions into one variable involving service at a given stage, we limit the number of random distributions that have to be specified and thus reduce the amount of field testing and verification that would be required to justify our choices of distributions. Secondly, by eliminating excessive detail we not only have a clearer picture of our true problem, but the run time for our simulation (if one is needed) is significantly reduced. That is, if we were to use a computer programming language such as SIMSCRIPT we would find that we could simulate on the order of a complete day (24 hours) of activity in only



one minute of computer time versus something like the DYN-TACS model which takes two or three times longer than real time to replicate one particular mission. Thus, the queueing approach allows us to reduce the immense time requirement associated with some of the sensitivity analyses mentioned earlier in this chapter and thus permits us to investigate the results of varying assumptions much more freely. Third, by not specifying current operations in excruciating detail, we avoid the pitfall of suboptimization and reduce the probability of overlooking alternate courses of action for improvement. That is, if we were to concentrate on all of the small details involved at each stage of service, even assuming we did all of the necessary verification of response times and their associated distributions, the best that we could hope for would be a model that fully duplicated only the present system. At the same time, however, the more complicated model would require substantial revision in the event that a new procedure, such as in our refueling case, were to be introduced prior to our implementation of the study. Also, if such a complicated model were to be accepted by various airport planning organizations, it might cause them to overlook alternative ways of conducting operations and, while permitting improvement on current procedures by such means as reducing service times, new arrangements of workload or entirely new procedures might be totally missed due to excessive dependence on the detailed model and the concomitant requirement to change the model if a substantial revision of general operating procedures were to take place. The queueing approach, on the other hand, permits the decision maker to select whatever means he feels will achieve the greatest level of improvement without worrying that if he changes details of the particular procedures of a given service stage that he would be forced to either rebuild his model or disregard its results as no longer fully applicable.



## B. QUEUEING THEORY APPLIED TO THE FIELD ARTILLERY SYSTEM

In the last section we saw how a typical queueing system approach could be used to analyze a rather common problem involving the scheduling and servicing of aircraft. Now it is time to address the question as to whether or not this technique can be expected to yield worthwhile results if applied to a combat process, in particular, to an analysis of the Field Artillery direct support system. In fact, the answer to this question is a resounding affirmative. Indeed, the choice of the airport example was made with an ulterior motive in addition to the obvious intent to provide an illustration of a typical queueing application, for its similarity with the direct support system is quite close although we shall have to do a little cutting and trimming in order to fit the example more exactly.

To begin with, we can regard the arrival of targets to the direct support system as being similar to the arrival of aircraft to the notice of the personnel in the airport control tower. Just as the personnel in the tower have no influence on aircraft which are not picked up on the tower's radar, or which do not make some communications link with the tower, likewise undetected targets on the battlefield are not subject to being fired upon by the artillery system, at least as long as they are not in close proximity to identified targets. Additionally, targets which cannot be fired upon prior to their moving away, or in some other manner avoiding artillery fire, can be compared with aircraft that are detected but have to make emergency landings. In this case, however, the surviving enemy target is the disaster analagous to the crash of the aircraft whereas the enemy unit which is taken under fire by some







other friendly element takes the place occupied by the plane which manages to make a relatively safe emergency landing. Preferably, in either case, we would have been able to give "adequate service" to the customer and thereby avoid disaster.

To continue the analogy, we observe that each plane can land on one of two runways in order to reach the terminals just as there are two main communications links to the firing batteries: wire and radio. Since we usually have three batteries per battalion, we likewise notice that in our example we have three separate terminals for the aircraft. When the planes reach the terminal for their three stages of service (unloading, refueling, reloading), it is evident that the length of time that each of these operations takes depends on such factors as the number of passengers aboard and the type of baggage and cargo being carried; how much fuel needs to be replaced; etc. Similarly, each target has certain characteristics which help to determine the total number of rounds necessary to inflict a given level of damage; errors in the Forward Observer's target location affect the time required to adjust onto the target and then enter fire for effect; etc. However, whereas our aircraft merely goes through its three service stages once per visit to the airport and is then ready for take-off, our artillery target can be recycled through the Fire Direction Center, the Firing Battery, and the Forward Observer segments several times. Finally, we come to the last stage where the aircraft departs the field after interacting with the tower in the guise of a third server. Since we do not require this centralized control from battalion headquarters in order to end a typical mission, our target would automatically depart from the system upon completion of the final rounds and the observer's assessment and end of mission message. Thus, we can modify our description of the airport in Fig 3, as



presented earlier, if we change control tower to battalion FDC, terminals to individual batteries, with the stages of service unloading, refueling, and reloading becoming the battery FDC, the firing sections (howitzers), and the forward observers, respectively (see Fig 4). In addition, we must also permit a recycling through this service system until the end of mission message is received, whereupon we cut off the third server and allow the targets to depart directly. Having thus illustrated the basics of the queueing approach and its advantages over some current methods, the next chapter will concentrate on the details of the Field Artillery queueing model and some of the specific factors of interest as mentioned earlier.



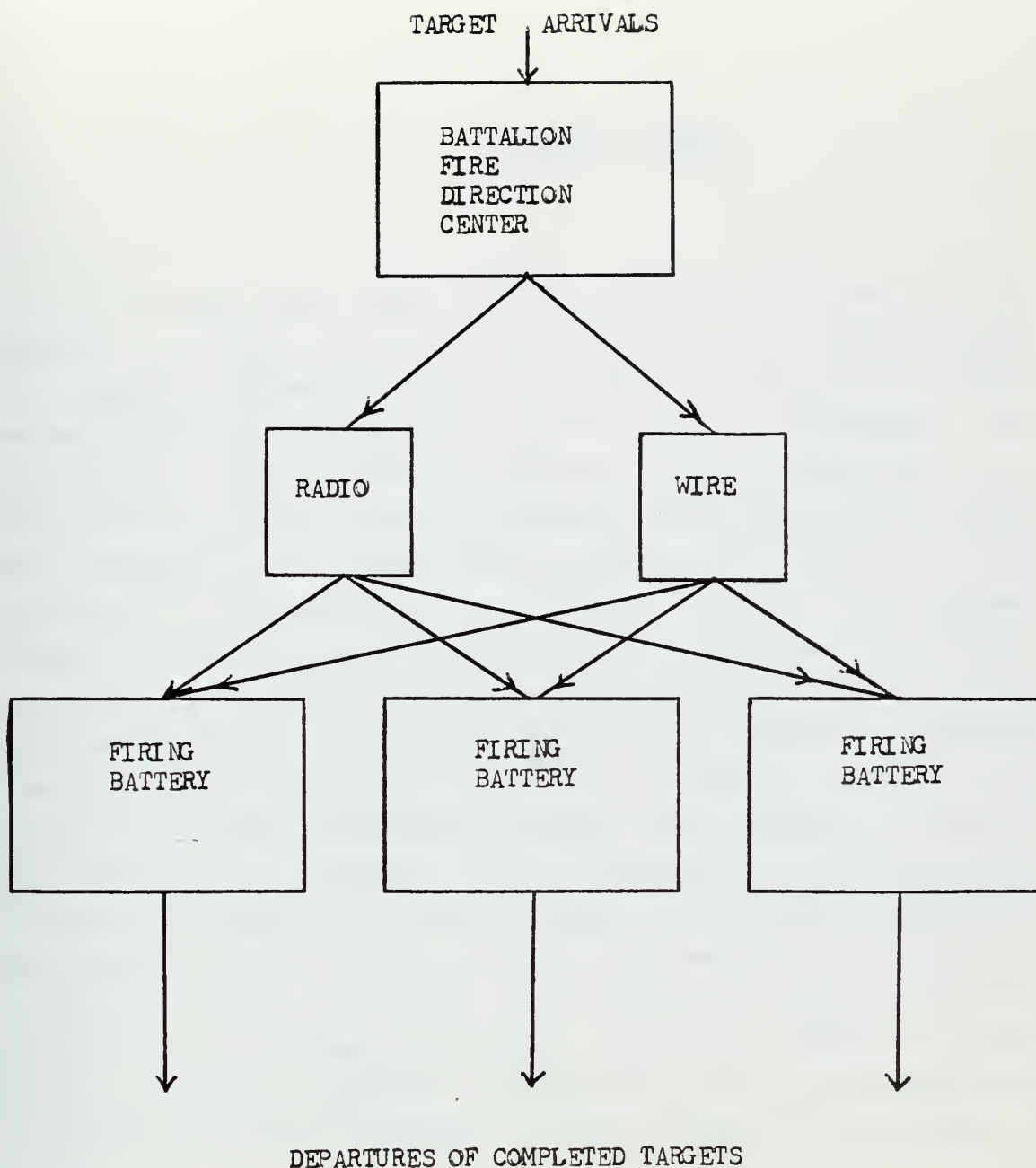


Figure 4 - THE ARTILLERY MODEL



## VI. A SPECIFIC MODEL

As we saw in the last chapter, it is a relatively simple matter to describe the Field Artillery Direct Support system in general queueing terms. However, except for an easily understood description, what do we gain by assuming this particular view of the process, as it pertains to our capacity to analyze Field Artillery procedures and problems? Two benefits are immediately apparent. First, as noted earlier, it is possible to use general service or response times for entire stages of the process, i.e. we do not have to specify, in excruciating detail, every sub-task in any particular phase. For example, we may realistically combine the activities of fuzing projectiles, loading them into the howitzers, and subsequently firing the weapon all into one distribution to represent the activities of the Firing Battery. Then, if we should discover through our analysis that the solution to our original problem is in the firing battery process, and not in another part of the overall system, we can either build more detailed models of that single service element or do further field experimentation as needed, without changing any other parts of our model.

The second benefit is derived from the fact that queueing theory is already a well developed academic discipline and thus we should hope to obtain some readily usable formulas to assist our analysis, much in the manner that we freely go to tables of integrals and other complicated functions when we have need of them in a particular problem. That is, we may make use of work which has already been completed and merely adapt it to our own purposes without the necessity to re-derive all aspects of





the subject. Thus, combined with the simplified description of our system, this last benefit may be extremely useful in permitting us to isolate the critical features of our process and see their long-range effects by the use of formulas already available. Indeed, since many specific results in queueing theory are highly dependent on the nature of the random distributions involved, we may also expect to estimate or measure some of the critical effects of making various distributional assumptions, the importance of which was noted in the introduction to this paper.

As we begin to put some flesh onto our skeletal queueing model, the first question that must be addressed is one of general notation. Throughout the remainder of this paper I shall use the common practice of referring to different queueing systems, either our current model or others mentioned only for comparison purposes, using the A/B/m/K/M method of notation. For any reader unfamiliar with this particular labeling technique, reference may be made to Appendix A, where I have included a brief description of the method along with a list of common abbreviations often seen in queueing literature.

#### A. A GENERAL FORMULATION: G/G/M

Since the ultimate goal of the present analysis is to determine the effects of different distributional assumptions when used in the model of the direct support system, our first attempt to parameterize the model should be as general as possible in order to avoid creating special cases as long as we can feasibly gain information without such assumptions. At this point, the most general model that could be employed is the G/G/m system. Here we are allowing both the arrival and service distributions to



remain unspecified, as well as the number of servers. We are also not placing any limits on the storage capacity of the system, nor in any way restricting the "customer" population. As is quite evident, all we have managed to do so far is to give a formal name, or description, to our skeleton of a model. Indeed, this most general description merely restates, in formal queueing terminology, the nature of the rather vague model which we have built earlier and are attempting to analyze. In order, then, to become more specific, and to make our system amenable to further analysis, let us first examine the two descriptors which we have not even attempted to write down, i.e. the storage capacity and population of the overall system.

#### 1. The Target Population

While the targets which could be acquired by our Field Artillery system are by no means infinite, as our shorthand notation seems to imply, it is also quite clear that we are not dealing with a closed system either. That is, while we certainly have some upper bounds on the number of targets which the enemy could field against us, we have no assurance that any specific mix would be placed in the fields of fire of our artillery units at any given time. We would hope, however, that battlefield intelligence might provide us with rapid and reasonable estimates of enemy strength and force mix. Nevertheless, for analysis purposes, we cannot possibly enumerate and investigate all feasible enemy mixes, especially when we consider that we must also allow for non-existent targets. By this last point, I do not necessarily restrict the definition to either false alarms or new weapons currently on the drawing board, although these are distinct possibilities, but, from the viewpoint of those in the artillery system who are the servers, i.e. the battalion and battery personnel in



particular, any fire mission expends their time, energy, and ammunition. This is the case even if no destruction of enemy forces takes place as a direct result of their firing. For example, a common technique used by maneuver units is to initiate "recon-by-fire" missions. In these instances, a target is called in and rounds are fired into an area where the enemy is suspected of being, even though no firm intelligence may be available to reinforce this belief or cause our attention to be given to a specific locale. The rationale behind the procedure is that, if the enemy is present, the incoming fire may cause him to respond and reveal his presence, either by returning fire in the belief that he has been fully discovered and is under attack or, alternatively, by causing him to abandon his position in favor of a safer location and thereby reveal himself to our forces. Since there always exists a significant probability that the enemy is not present, or will not react to this probing technique, it is quite clear that this is not a target in the regularly understood sense of the word. Yet, all of the components of the Field Artillery system are functioning as if there were a target present and thus, to the degree that they are employed upon this mission, they are unavailable for missions of equal or lesser priority where the enemy might actually be present and more liable to damage.

In addition to these considerations, let us also look at the manner in which DYN-TACS and the AMMO RATES model handle this problem. In the former model, targets are only generated upon detection by friendly elements and are confined exclusively to the enemy forces present on the localized battlefield. No attempt is made to include special missions, like recon-by-fire, or even competing missions designated for firing outside of the immediate area of conflict. Thus, only those missions generated directly by contact with opposing forces are played. While these







missions are usually of high priority, and thus might preempt other fires, they certainly are not entirely representative of the full capacity of the Field Artillery system. On the other hand, the BAM routines of the AMMO RATES model do not account for ammunition fired in close support (as does DYN TACS), but only for more general missions, such as fires on detected enemy units which are not involved in close combat. Estimates of artillery utilization and ammunition expenditures due to missions called in by maneuver units must be played separately. Thus, we also see in this case that the full spectrum of artillery fires are not played by this model either.

One of the major reasons for this apparent disparity in accounting for total artillery utilization is due to the differences in scale of resolution of these models as was described in Chapter IV of this paper. Indeed, we must recognize the fact that each of these models fulfills a definite purpose in the general attempts of the modelling community to gauge weapons effectiveness on the battlefield and in predicting bounds for our ammunition stockpiles. However, due to their complexity and particular concentration on separate aspects of artillery employment, neither of these models is sufficient in providing insights regarding the overall workings of the Field Artillery system. Since we can reasonably expect, and our current doctrine reinforces this view, that when not engaged in supporting maneuver units, our artillery weapons will be employed to bring fire upon other enemy targets which are being located by the vast array of sophisticated sensors and other techniques now available, it is clear that we need an overall model with which we can examine the effects of this type of utilization without being restricted to any specific level of the battlefield to the exclusion of other areas. Thus, we see the importance of not placing any limits on our target population, thereby leaving our system with the



capacity to fire upon whatever enemy elements which are detected and within range of our fires. By permitting our model to handle all possible categories and numbers of targets which may become available, we can begin to evaluate the effects of different decision rules regarding the assignment of target priorities within the overall context of the general artillery system.

## 2. Storage Capacity

It is quite clear that, just as our target population is not without some upper bound, the ability of the Field Artillery system to store targets also has practical limitations due to the number of storage locations in the FADAC computers at both battalion and battery level, as well as the limited amount of paper and other manual recording devices within our units. Nevertheless, for the purpose of our general model, we choose not to specify any exact limit on the system's maximum capacity. There are several reasons for this approach. First, and perhaps most importantly, we have no solid combat data which will give us any firm indication of what this upper bound might actually be. Secondly, the number of targets that will occupy the queue in a waiting status will, to a certain extent, depend on the type of queueing and serving disciplines which we later impose on our G/G/m system, as well as on the arrival rate of targets into the overall system. These two factors alone are sufficient to argue strongly for not placing any upper limit on storage capacity. However, two other arguments can also be advanced which must eventually be considered in our analysis, so we might as well address them here. That is, if we are attempting to examine the effects of different priorities being assigned to different targets, we would want information about the number of targets which are delayed due to their being of lower priority than others



and also how much of a delay is imposed would also be of interest to us. However, with a fixed storage capacity system, we would be forced to eliminate targets from our model once we have filled the queue spaces. While we could still manage to process the higher priority targets, by removing and discarding low priority targets from the full queue to make room for new, higher ranked targets, we would still have difficulties confronting our analysis. For example, if some of the high priority targets are very mobile, we have the distinct possibility that even by putting them in the queue ahead of other targets, we may nonetheless not finish service on an equal category target in time to fire upon the mobile one before it has moved away. Thus, by the time we draw this new target from the queue, we may have to discard the mission due to the target being out of range or otherwise no longer available. In addition, we might have had to discard from the queue a stationary, but lower ranked, target in order to make room, and thus we can no longer fire on this one either. However, we can easily handle this problem by keeping all arriving targets in an unbounded queue and make our decision on target availability as we draw the next one for service after completing a fire mission on earlier arrivals.

This concern with keeping track of individual targets now brings us to the other additional reason for not limiting our storage capacity. Namely, if we desire detailed information as to the number of targets of varying priorities, and other descriptions, which are forced to wait for service, we soon will require the use of a computer simulation to answer our questions since the exact waiting periods of the different categories of interest will be highly dependent on which targets have preceded the latest arrivals into the queue. In designing such a possible simulation, if we decide to have a fixed storage capacity system, it would be necessary to have additional subroutines





to scan the queue whenever a new arrival occurs and the queue is full in order to determine which target should be discarded in order to provide room for the new arrival. This would not only require us to program maximum waiting times for various types of targets but, in the event that this maximum time had not yet been exceeded when a new arrival occurred, we would need to estimate the remaining time for the current mission in order to evaluate which target should be discarded. Since different target types would have different mission requirements, this would necessitate several additional routines to estimate the residual life, or forward recurrence time, of the current mission.[Ref. 15, Vol. 1, p. 169-174] Fortunately, we can avoid all of these extra complications by providing an unlimited queue and simply determine whether a target has been delayed too long when we draw it from the queue and prepare to fire. If our waiting time has exceeded a user specified limit, we merely eliminate that target and draw the next in line and perform the same test until we find one which can be fired. Thus, the advantages of using an unlimited storage system are quite important to us and clearly, without any concrete evidence to the contrary, we should opt for such a discipline until we discover valid reasons to change our model.

#### B. HOW MANY SERVERS AND WHAT LEVEL OF RESOLUTION?

Since we have been examining the aspects of our queueing model in stages from the right side of our shorthand notation, let us continue to do so and now address the questions of how many service elements we should include in our model formulation and at what level of resolution. In Chapter III reference was made to the fact that the battalion FDC assigns missions to the individual batteries





based on such factors as which of them are already engaged, ammunition availability, etc. With this in mind, we see that we can choose several possible arrangements to use within our model. First, we could decide to examine our system on the basis of one server being the equivalent of one battalion, and thus each of the operations that take place on the firing battery level would be sub-stages of service and our system would appear as in Fig 5. Here, our arrival stream would consist of those targets which are directed through the battalion FDC and then on to the appropriate firing batteries. On the other hand, we could also examine the distributional effects of interest by permitting target arrivals to proceed directly to the batteries, even though we know that a great many also pass through the battalion echelon in their course of processing. This latter procedure is already used in both the DYN TACS and AMMO RATES models and would indeed seem to be our best choice. By examining the battery directly, we are looking at the smallest self-contained firing element in the Field Artillery system, since each battery has its own FDC and radio nets with which to process missions, as well as the weapons themselves. Indeed, since the battalion serves primarily as a coordination and administrative center, we recognize that if we were to look at the assignment of targets from the battalion FDC we would find that they are rather evenly apportioned to the individual batteries in order to insure an equality of workload. This equalization of work effort is employed, not just from a desire to distribute the missions "fairly" from some philosophical point of view, but also in an attempt to equalize ammunition resupply requirements and to guard against the possibility that any individual battery might become too fatigued and/or depleted in ammunition to the point that it would be unable to continue its primary mission of support even if it were the only one otherwise available. That is, by equalizing the assignment of missions, we serve to guarantee that,



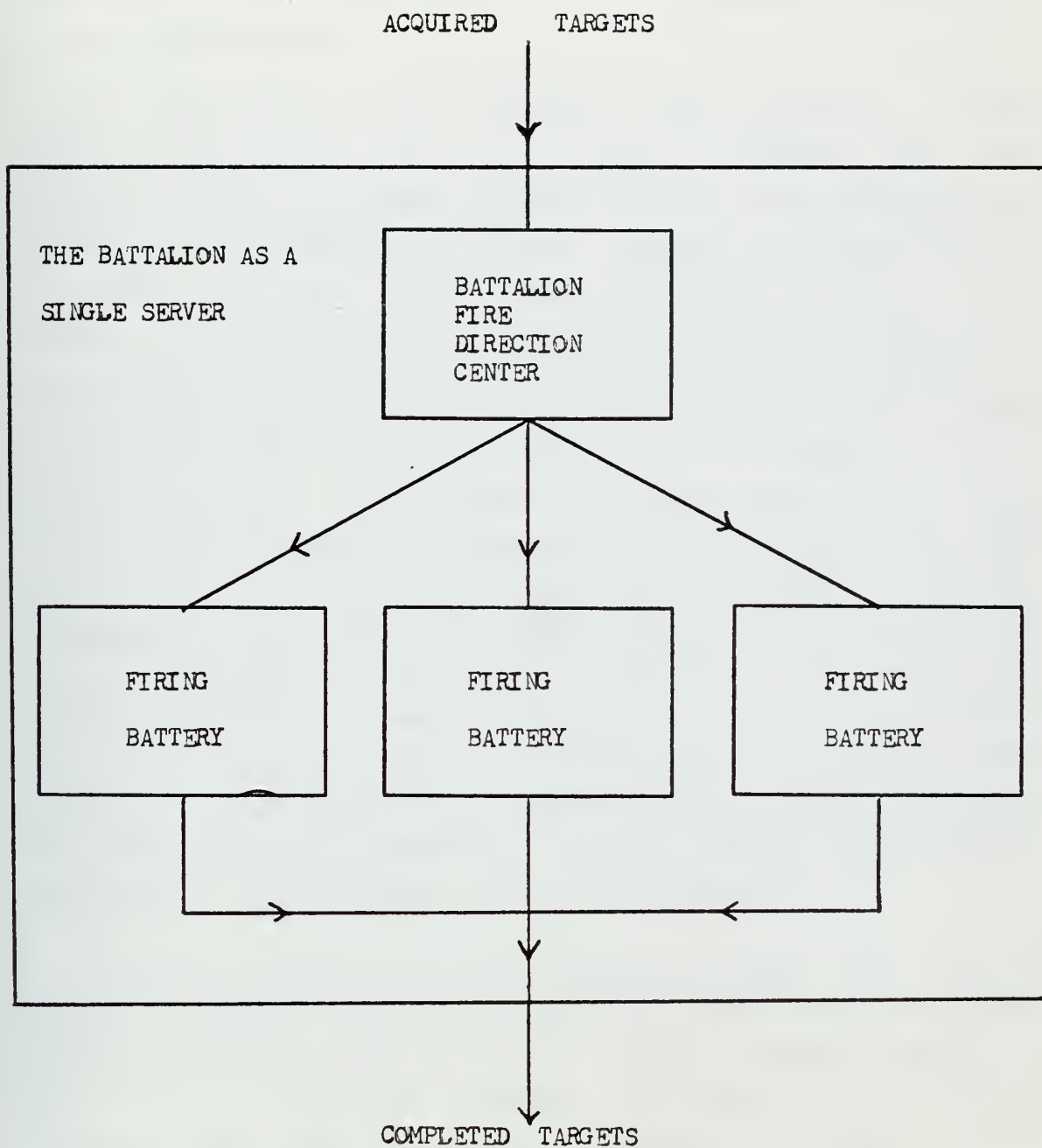


Figure 5 - A BATTALION LEVEL VIEW



unless combat reaches such an intensity that all our units run out of ammunition, there will be at least one battery available at all times which is fully capable of responding to new requirements.

In addition to our "equalized" work argument, we must also recognize that current doctrine provides for fire missions which may short-circuit the battalion level and go directly to the firing batteries. Perhaps the most familiar of these is that group known as Final Protective Fires (FPFs). In instances when friendly units are coming under heavy attack and need a high volume of supporting fire to avoid being overrun by the enemy, the maneuver unit calls directly to its assigned supporting artillery unit and requests its FPF. This request is sent directly to the firing unit because any delay would be critical and, upon receipt, the battery initiates this particular mission, preempting any other fires that may have been scheduled or in process when the call arrived. If we do not model our system directly at the battery level, then we must provide some additional equations or subroutines to permit these types of interruptions and thus we would unduly complicate our model when it is possible to avoid these complications by working at the battery level to begin with.

However, perhaps the most telling argument for battery level treatment has been provided by the Commandant of the U.S. Army Field Artillery School, Major General David E. Ott. In several recent articles[Ref. 21, 22, 23] concerning the Field Artillery's attempts to adjust to the changed nature of modern combat, especially as it is perceived after the 1973 Mideast war, and the introduction of Anti-tank Guided Missiles (ATGMs) in massive quantities on the battlefield, MG Ott has written:

We intend to "dedicate" field artillery units to maneuver elements moving to contact. By this, we mean that a battery from the direct support





artillery battalion can be dedicated on a one-on-one basis to a leading maneuver element - perhaps a company team - to answer immediate calls for suppressive fire. It will be the maneuver commander's choice as to which of his elements will receive this dedicated support. We visualize that a direct support battalion could provide up to two batteries in this dedicated role - keeping one free for quick response to other elements of the committed force.

A dedicated unit will "monitor" the command frequencies of the supported maneuver company for the express purpose of following the tactical situation and answering immediate calls for fire from a particular maneuver element. This will allow infantrymen and tankers to call for fire in emergencies without changing frequencies...

Infantry and armor captains, lieutenants and platoon sergeants will be taught a simplified system for calling for and adjusting suppressive fires since we acknowledge that the artillery forward observer will not always be in a position to call for instant artillery fire throughout the company sector.[Ref. 22, p. 52]

Thus, with a greater number of calls for fire being sent directly to the battery and short-circuiting traditional FDC procedures, the argument for considering the battery level as the primary element of our queueing model becomes irrefutable.

Now that we have settled on the level of resolution, the next pertinent question is: how many batteries should we use in our formulation? As MG Ott has stated, two thirds of our direct support units will probably be employed in the "dedicated" role. When we consider that about 75 percent of all U.S. artillery is used in direct support, we see that roughly half of all our Field Artillery units will be "dedicated". This also does not preclude the possibility that the remaining DS units will also be used in a dedicated role, merely that we aren't starting out with all of our forces entirely committed to one particular type of mission. Because of this new emphasis on dedication, we observe that, while artillery units will still be emplaced so as to provide overlapping fields of fire, the primary emphasis will be on the individual battery and for this reason I have



chosen to analyze a single server system. Thus, our  $G/G/m$  model has now become a  $G/G/1$  system.

### C. DISTRIBUTIONAL CONSIDERATIONS

Having examined three of the standard factors used in our shorthand notation for queueing systems, we must now address the question of what distributions will be used within our model. This brings us directly to the problem of analyzing our model of the  $G/G/1$  queue. However, we now find ourselves confronted with a serious problem, since even such a simple item as the average waiting time for this system is unknown! Indeed, while certain highly mathematical methods can be used to obtain various solutions to the  $G/G/1$  system, they are heavily dependent upon the use of Laplace transforms and "spectral" methods of solving what is known as Lindley's Integral Equation.[Ref. 15, p. 273] As if this series of mathematical difficulties were not enough, it turns out that we must have precise information about our specific process in order to un-transform our basic solution back into the realm where we can interpret our results. This means that, while certain forms of a general solution are left in transform format, in order to have any validity for our study we must know the actual equations of the arrival and service distributions in order to be able to perform these reverse transformations. In addition, there is no guarantee that any particular equation will have a closed-form solution. Thus, it would appear that, in order to gain any further headway, we must begin to make assumptions about the nature of either the arrival or service distribution (or both). Unfortunately, it is just this facet of our problem which we wanted to study in detail by using previously developed formulas to evaluate varying distributional assumptions. As was pointed out earlier in



this chapter, we would refrain from making any really critical assumptions until forced to do so by the developing nature of our problem and it would appear that we have now reached that particular decision point. The basic question now becomes, which do we modify, the arrival or service distribution? Let us examine the important aspects of the problem affecting our choice and see if we can arrive at a reasonable approach to continue our analysis.

Clearly, whichever distribution we choose to fix by our assumptions, we desire that further changes or assumptions should be kept to a minimum. In addition, we do not want to eliminate our analysis project by virtue of the assumptions which we will proceed to make at this point, but we must still find some way to obtain useful formulas as well. Therefore, in order to resolve this dilemma, let us leave our model for a short while and concentrate our attention on the actual military problem. It is quite evident that, as targets arrive and are fired upon, we have varying degrees of control over the process. That is, while we can implement changes in our decision criteria and in our procedures at the Forward Observer, FDC, and Firing Battery elements with relative ease, we have virtually no control over the target arrival process since the timing of such arrivals, as well as the varying types of targets, are highly dependent on the force mix which the enemy decides to deploy against us. Since we have a very limited span of control on the arrival process, if indeed we can be considered as having any control at all, it would be fruitless to examine various distributional assumptions regarding the arrival process since there is no way we could modify it if our study results pointed to that end. Indeed, since the acquisition of targets is also highly dependent on the types of detecting devices we use, it would also be fruitless to try to choose one particular set of arrival distributions based on current technology as well as enemy force mix.





Therefore, it appears that the most favorable course of action would be to make an assumption about the nature of the arrival process and still leave ourselves free to test various response distributions and their effects on our modeling capability. In addition, since the table of CLGP assumptions presented earlier indicates that many of these service factors have not been studied in detail, it is possible that we may be able to isolate those aspects of artillery response which require closer scrutiny in order to come to a more satisfactory way of modeling combat. If, on the other hand, the net results obtained from our examination of different distributions are sufficiently similar, we may be able to conclude that further study of this aspect of combat is not required and thus permit more effort to be addressed to other items of concern with a high degree of assurance that our current modeling techniques are sufficiently accurate for analysis purposes.

Even with these considerations in mind, we must, nonetheless, still choose a particular distribution to use in the arrival process in order to be able to continue our evaluation. As may be noted in Appendix A, there are several possibilities which are quite common in practice, not to mention others which appear less frequently. Fortunately, however, both queueing and renewal theory have what amounts to a form of Central Limit Theorem similar to that frequently called into service by statisticians. Whereas the latter can make use of the Law of Large Numbers to postulate that, in the long run at least, many numerical techniques tend toward a Normal distribution, likewise Alexandr Khintchine has shown that in many cases the sum of a large number of independent renewal processes, each with an arbitrary distribution of renewal time, will tend toward a Poisson process.[Ref. 14, p. 23-36] While we would like to change our model from that of a G/G/1 system to a new M/G/1 version by recognizing that a Poisson arrival process





is equivalent to having exponentially distributed interarrival times,[Ref. 26, p. 120-121] we must first try to ascertain whether we can safely adopt the Poisson assumption or whether the available evidence would indicate that another distribution would be more appropriate.

If we were engaging in our analysis merely as an academic exercise, rather than hoping to obtain a useful method of evaluating distributional assumptions, it would probably be sufficient to point to other studies that use some version of a Poisson arrival process to justify our own usage. In that case, we would merely cite the CLGP Communications study previously referred to and continue our examination. In addition, it can be shown, using a simple proof by analogy[Ref. 5, p. 958-964], that the routines used in the AMMO RATES model to assign detection times to acquired targets results in an overall nonhomogeneous Poisson process, although the actual documentation of that model does not indicate that this was the original intention of the designers but came about as a result of the allocation techniques which they used. Besides these other models, however, we would like to feel confident that the combat process itself supports our adoption of the Poisson assumption. While very little empirical evidence exists to support any particular choice of distribution, perhaps a heuristic argument will serve the purpose, especially in view of the other arguments given above.

Turning once again to the battlefield, we readily observe that the target acquisition process, which produces the arrival stream of our model, has different characteristics depending upon the types of forces engaged and the variety of additional sensors that may be employed by both sides. That is, there are certain basic functions which are performed differently by various observers and sensors which produce the acquired targets. For example, if



we look at a friendly infantry platoon in a defensive posture, we discover that, in addition to the troops in their fighting positions, we also have the capability to employ various types of sensors and detecting radar to assist in detecting any approaching enemy forces. It is clear in this example that if we employ a ground radar, we will have a certain distribution which will describe the manner in which moving enemy targets, such as tanks and APCs, will be detected, but other types, such as stationary bunkers, will only be acquired by visual or other means, certainly not directly by the radar. Thus, it is apparent that we have a different acquisition process in operation for each type of sensing device (including each individual soldier) employed on the battlefield.

In addition to this multitude of possible detectors, we also note that we are increasing the number of channels by means of which target locations can be brought to the attention of artillery units, especially by using the dedicated battery technique, which calls for our artillery units to directly monitor the supported unit's radio net in order to respond instantly. In addition to this summing of individual renewal processes for each sensor, we also note that the probability that any one person (or sensor) detects more than one target at any particular instant is quite small, since his attention or capability will focus on his first detection and act as a filter which limits his field of vision to a much narrower area. Thus, targets which are in a dispersed formation will not all be detected by one individual or device. This further aspect of the detection process would also seem to support a Poisson assumption since one of the basic properties of such a process is that the probability of two events occurring in an extremely short period of time be quite small.[Ref. 26, p. 118] Thus, we see that there are other factors operating on the battlefield which tend to support the Poisson process



assumption in addition to the limiting argument originally developed by Khintchine and with this in mind we shall accept this modification and change our model to an M/G/1 system

#### D. THE FINAL MODEL

In this chapter we have developed our model beyond the point of mere generality, from a basic G/G/m system to a final M/G/1 version. While we have been forced to make several assumptions regarding such factors as the system's storage capacity and the size of the target population, we have attempted to justify our approach, not only on the grounds of modeling simplicity, but also by a close examination of the actual combat process which we are attempting to analyze. In this respect, perhaps our most critical assumption in the final model is the Poisson character of the arrival stream. However, even here we have stressed not only the need for some type of further simplification to render the model more tractable, but we have also shown the applicability of a central limiting argument which is firmly supported by the actual combat interactions that we may expect on present and future battlefields. At this point, in order to continue our study, we must address the questions of what information we hope to measure in our particular model and how this translates to significant aspects of the combat process. Since this is the whole question of what Measures Of Effectiveness (MOEs) to use, we shall address that problem in detail in the following chapter.





## VII. MEASURES OF EFFECTIVENESS

### A. GENERAL CONSIDERATIONS

The most critical aspect of any Operations Research study is concerned with the choice of the correct, or most appropriate and applicable, set of Measures of Effectiveness (MOEs). By our choice of what to measure and interpret, we influence the final conclusions of any particular study and, in some cases, determine the outcome long in advance of the detailed analysis. A classic example of just how critical the choice of MOEs can be is presented in Morse and Kimbal's Methods of Operations Research in regard to evaluating the value of anti-aircraft weapons in protecting Allied shipping during World War II.[Ref. 19, p. 52-53] While this example occurred over thirty years ago, the common pitfalls it discloses are still of great danger to any analyst confronted with a practical problem and it behooves us to take some time, prior to developing complex formulas or cranking out a massive volume of numbers, to decide just what we wish to measure with our model and how we can interpret the results and their impact on battlefield activities.

It has already been noted earlier that most major models currently in use by the analysis community are designed to be predictive in nature, especially due to the requirement for cost-effectiveness studies for different weapons alternatives. Not surprisingly, therefore, we find this emphasis cropping up in the choice of MOEs which are made by



the planners and analysts involved in obtaining information from these models. Thus, the most frequently used measure is the number of enemy personnel and major pieces of equipment (usually tanks and APCs) which are destroyed by the friendly forces by some time period in the battle. This is not to imply that these are the only numbers which are printed out by the models, but they are the ones used for the final justifications of the study's conclusions. Thus, while both DYN TACS and AMMO RATES, for example, both give an immense volume of additional statistics, such as vehicle velocities and locations at various times during the battle (DYN TACS) or the number of rounds fired (AMMO RATES), we still concentrate our attention on the measures of enemy casualties and usually look at these other numbers only in the event of tied outcomes.

In essence, then, what we have done in constructing and using these and other models, is to rely heavily upon the pure engineering aspects of current and proposed weapons systems as they fit into our tactical doctrine. That is, while we don't merely line up a series of targets on a practice range and see which weapon produces the greatest amount of damage, we attempt to place this damage into the context of a military operation. At this point, however, we are faced with some grave difficulties. For example, if we desire to test different types of artillery ammunition, each with a different size lethal area, we can first do our basic development tests prior to the COEA. At that time we will get some relatively firm information as to the exact size of the danger zone and, for the purpose of this example, let us suppose that the first shell has a lethal area which includes all unprotected personnel within five meters of the impact with a guarantee of achieving 100 percent casualties. Let us also assume that the second round has a radius of destruction equal to ten meters and similar damage characteristics within that zone. From this, it is quite



clear that if we merely line up some closely grouped targets and fire different rounds at them, the second shell will cause the greater amount of destruction. However, we realize that such groupings of personnel might be highly unlikely in actual combat and so we input the proper descriptors of the ammunition performance into a combat model and attempt to determine which type of ammunition is more effective in an operational environment. At this point, however, our basic choice of model and input enemy force mix almost completely determines our final conclusions unless some totally unforeseen interactions take place. For example, if we use our two proposed types of ammunition against a heavily armored force, neither will appear to do very well since we are discriminating among them on the basis of their anti-personnel capability. On the other hand, should we use them against a predominantly infantry enemy force, the size of the enemy units which are input to the model and subsequently fired upon will drive the final conclusions of the study. If the opposing force attacks or maneuvers in very tightly organized groups we are likely to see that, if the groups themselves are large, the second round will have the greater effect while, if the groups are small, the first round will be more effective. On the other hand, the model might use the same size groups for different runs but vary the dispersion pattern within each group in such a manner as to effect which types of ammunition will be considered most effective. Since the emphasis is purely on enemy casualties obtained, it is possible to estimate the winner of our competition solely by a close examination of the model's inputs and casualty routines.

However, this still does not really solve our problem of determining true effectiveness, since it is quite possible that different practices of weapons employment might have vastly different implications for our capability. Thus, we might feel that the first ammunition type would be most





effective in use against units close to the Forward Edge of the Battle Area (FEBA) since the proximity of other weapons adds to the basic military requirement to remain relatively dispersed in order to avoid excessive casualties. Yet, if our weapons also have the range capability to reach out behind the FEBA and strike enemy assembly and staging areas, the second round will be more effective in those circumstances because of the greater proximity of personnel to each other in the more secure areas. In addition, we must realize that any weapon development simultaneously starts work on counter measures and this example is no exception. Thus, our opponent might decide to increase the basic armor protection of his infantry, or move his staging areas further to the rear and out of our range, or finally, he might disperse his forces even more widely in order to reduce the effectiveness of whichever type of ammunition we adopt. Unfortunately, our models do not play this adaptability on the part of our opponents, nor do they give any firm answer as to which of these alternatives (or combination thereof) he might adopt.

Finally, especially when we are considering artillery systems, our current models do not play the occurrence of false detections and/or erroneous identification. For example, in DYN-TACS, once an enemy target is acquired by a firing element as a firm detection, there is no doubt about the true nature of the target. That is, there are no mistakes in identification: if an APC generated the detection, there is no possibility that it might be mistaken for a tank, or vice versa. In addition, only the actual presence of enemy units can bring about a detection event, which is not entirely correct as we noted when discussing the recon-by-fire technique which is often employed when it is not feasible or desirable to actually send troops into an area to determine whether the enemy is present. Thus, we see that in a situation where we are attempting to model





artillery fires, the problem of false detections and/or erroneous identification becomes quite important and if our MOE is based on actual casualties produced, we are failing to account for all of the rounds that would be fired. That is, since we only shoot missions that are directed against actual targets, we have no estimate of the rounds fired against non-existent but suspected enemy concentrations. While we would certainly hope that our vast array of sensor systems and intelligence personnel will provide us with accurate data regarding the enemy's true location and disposition, nevertheless, we must still recognize this important problem.

There is also another aspect to this matter which we touched upon in a previous section. That is, as long as we are firing, even if it be at non-existent targets, we are expending ammunition and effort and are not allocating our fires against what might be the best choice of targets available. Thus, in addition to underestimating the number of rounds fired, we may also be over valuing the apparent effectiveness of the rounds we do fire by not considering these false alarms. However, even without stressing this aspect of actual combat, we can see that current developments require us to consider more facets of our operations than just net enemy casualties. If we go back to MG Ott's article we find the following passage:

Finally, if necessary, we will give up some degree of accuracy in the interest of speed for immediate suppression. When maneuver elements come under fire, our reaction must be quick and violent. Two 155-mm rounds impacting 200-300 meters from an enemy ATGM gunner will surely cause him some concern, whereas a battalion firing three volleys on target 5 minutes later might well be too late.[Ref. 22, p. 52]

It is apparent that, as suppression and other non-casualty producing effects assume a higher level of importance in tactical operations, our concern soon broadens to include



factors such as the timeliness of our fires and how many targets we can shoot at within a reasonable period of delay for the purposes of data computation, etc. Also in MG Ott's article, reference is made to a more extensive use of smoke munitions which will have an effect on target visibility on the battlefield and it is quite apparent that the use of smoke, per se, is not likely to produce voluminous casualty figures due to the subsequent degradation of detection capability that will result, not to mention the fact that unless one hits an infantry soldier with the actual canister, smoke rounds themselves do not produce casualties. This is applicable, not only to the suppressed enemy force, but to our own troops as well since reducing battlefield visibility affects both sides. Thus, if we continue to place our emphasis on casualties, we are neglecting other pertinent military factors which would appear to be gaining in importance regarding their impact on ordinary operations.

While we are considering the problems associated with timeliness on the battlefield, we are also led to examine another aspect of the problem as concerns firing against moving targets. In the high resolution models (such as DYN-TACS), this factor is usually of minor concern since the model keeps track of the location and velocities of all maneuvering units through-out the course of the simulation and when fire is called for the model assesses the actual round impact for artillery weapons as well as the line of sight trajectory for direct fire weapons. However, in the lower resolution, more highly aggregated models (such as AMMO RATES), this degree of detail is not feasible and thus decision variables are put into the model which serve to indicate that if a mobile target cannot be fired upon within a certain user specified minimum time restriction, then that target is dropped from further consideration unless it is detected at a subsequent time. We have already looked at this problem when we were considering the size storage



capacity for our queueing model and we have no need to reiterate all of the arguments advanced in that section. However, one point should be made here concerning that problem and how it affects our consideration of MOEs. That is, upon what factors do we base our choice of minimum delay times, after which we cannot fire at a moving target? Close examination of this point reveals that, while some data is obtained from other, higher resolution models, by and large the numbers which are input for this decision point are based on the old standby "military experience and professional judgment." Now, as a military man myself, I would be one of the last to scoff at or denigrate this important factor in tactical and strategic decision making. However, we must also recognize that military experience, per se, is no guarantee that we will obtain the correct conclusions or insights, or even make the proper estimations. Indeed, this is clear if we look back into history at past wars and their effects on doctrine and weaponry. For example, many nations experienced the slaughter of trench warfare in World War I, yet it was primarily the Germans who fully developed the counter to that stagnation which we still call "blitzkrieg." Yet, the other nations which participated in that war, and also gained military experience, did not always come to the same set of conclusions, to wit: the Maginot Line. Thus, if military experience is not infallible, one of the considerations for our MOE might be to determine if we can set some practical limits on these delay times for acquired targets and measure the effect of different cut-off points as regards optimal artillery employment.

#### B. SPECIFIC FACTORS OF CONCERN

Having thus demonstrated the importance of factors





associated with the timing and allocation of fires, we must now formulate the questions which we shall try to answer by using our specific queueing model. Then, in the following section, I shall present the specific measures which our queueing approach provides and which are applicable to our analysis. Using this procedure then, we note that we must address the following questions:

1. What percentage of the time is our battery busy?
2. If the battery is busy when we randomly check the system, how long will the current mission last before being completed?
3. How long is the typical busy period and how much variability do we observe?
4. How many targets will be served during a typical busy period?
5. How many targets can we expect to observe in the queue and the entire system if we take a random glance at the model?
6. What is the average service time and the average waiting time for targets?
7. Finally, given that we must wait, how long is the delay?

Since the posing of these basic questions, as well as the manner in which we attempt to answer them, will have an important bearing on our study results, let us examine each of them for their implications pertaining to actual combat operations. Thus, our first requirement is to determine the percentage of time that our unit is engaged in firing missions. While this seems to be a very elementary point it is, nonetheless, quite important for this measure will be one of the driving forces of our system's behavior. Indeed,



we would expect, even without examining the specific formulas which describe the behavior of our system, that if we are seldom busy then such items of interest as the average delay, etc. will be relatively small, whereas if our battery is almost constantly busy, new arrivals will have to wait for longer periods of time in order to be served. However, the utilization factor for our system is more important than serving as a major driving force for other system parameters of interest. In addition to that aspect of our problem, the percent of time the unit is found engaged in a fire mission has implications regarding the fatigue of our personnel and the amount of work that must also be accomplished by our supply system in providing adequate stocks of ammunition. That is, if we usually fire an average of ten percent of the day, this means that slightly less than three hours are taken up in firing and we are thus able to spend a good portion of our time on other tasks, such as improving the battery position, moving to new locations, or performing necessary maintenance without interruption of firing. On the other hand, if our battery is engaged for 80-90 percent of a typical day, this gives us very little opportunity for those other tasks mentioned above and would tell us that if we must stop firing for some reason that such a course of action might have disastrous consequences for units needing our fire support. Thus, while our percent of time busy is a relatively simple value to compute, it has enormous implications for our tactical procedures.

Our second question, relating to the remaining length of service for the particular mission being fired when we take a glance at the system, is important for a couple of reasons. First, it provides us with an estimate of how much more effort is required before we can plan on either relaxing into an idle status or, in the event that more targets are still to be served, how soon we can begin firing



the next mission. This estimate can be quite important, especially if the next target is of extremely high priority or is very mobile, since our estimated time remaining for the current mission will allow us to decide whether it would be more advantageous to interrupt the old mission to take the new target under fire immediately or whether we could safely finish our current task prior to firing the next target. This information on the residual life of a target can thus provide us with decision criteria as to which categories of targets deserve pre-emptive treatment and which can afford to wait. For example, by getting estimates of the residual life, we can then check other models which use a time limit on firing against mobile targets to determine if we are losing a significant number of possible targets due to excessively short waiting times and, if this is the case, we may either reevaluate the time limits we use or else allow these mobile targets to get special treatment by changing the handling procedures associated with this category target.

Intimately related to the first two questions are numbers three and four, since they pertain to the amount of work done in a typical busy period. By looking at the responses to these two questions we should hope to be able to tell whether we have many short periods of heavy activity or several longer periods of more moderate activity. This measure, and that associated with the average number of targets handled per busy period, will be able to give us some insight into standard battery activities and how they are affected by fire missions. For example, we have already noted the importance of battery utilization as regards such matters as maintenance, etc. By examining the factors pertaining to questions three and four, we should be able to evaluate whether or not we will have the opportunity to conduct short maintenance operations between missions or whether we will have to consider pulling different gun





sections off line in order to permit adequate weapon maintenance to be performed. Since this decision affects the number of rounds that will be fired, and thus, directly affects the amount of resupply activities we must engage in, not to mention the obvious fact of enemy casualties, we can certainly understand the need for some adequate insights into this part of the combat process.

The remaining three questions pertain to overall system performance rather than to activities solely related to firing periods. Thus, the number of targets in the queue at any one time is a measure of the backlog of work to be done and therefore we would hope that, if our system is functioning adequately, this number, and the associated average waiting and firing times, would be small. Yet, we must realize that the only way we will ever guarantee that no queue forms, and thus no delays for service, is to have a firing element, whether it be battery size or of some other configuration, always available in order to handle each new target as it arrives. This would be the equivalent of an M/G/ system and it is quite clear that our limited national resources will never allow us to achieve this unlimited number of firing elements to handle each new arrival. Thus, the average queue buildup will serve as a measure of how well our procedures achieve their desired ends of serving targets within short response times and will also provide some additional information, besides that obtained from the utilization factor, regarding how well our battery is performing its mission. For example, it would be possible to have a high utilization factor, such as 90 percent, without suffering adverse effects on our firing procedures. This could occur in a number of ways. Thus, we could have a series of arrivals that occur just after we finish a mission, thereby causing us to start firing again almost immediately, yet with very little queue buildup because the next arrival would also occur just as we finish with the



current target. While we would still be concerned about our ability to handle maintenance, resupply, and fatigue problems, this particular situation is certainly to be preferred over the case when we would have the same utilization factor but the targets are arriving into the firing system such that we cannot respond fast enough and thus the queue increases substantially in addition to the heavy utilization of weapons and personnel. It is obvious from this example that the utilization factor alone is not sufficient to fully describe the activities of interest and thus we are led to consider such other measures as the number of targets which are waiting for service and their associated average delay and overall service times.

However, before proceeding to present the formulas we will use to measure these items of interest, the last question in our list requires some further explanation inasmuch as we are already measuring waiting times as part of the response to question number six. The fact which we must recognize here is that the average waiting time for the system is different than the average waiting time for those targets which incur a delay in being fired due to other missions being in progress when they arrive. This is not as contradictory as it first appears. When we measure the average waiting time for the entire system we are merely recording the amount of time a target is delayed until firing on it begins. Clearly then, when a target arrives and no firing is in progress the waiting period until processing begins is zero. This accumulation of zero waiting times is included when we average the delays for all targets and serves to reduce the overall average delay. However, if we have a fairly high utilization factor, those targets which arrive while firing is already under way will incur larger delays and, if we restrict ourselves to the total system delay, we thereby underestimate the waiting times of the delayed targets as compared to all targets.



This last question (number 7) addresses this problem by measuring the delays only for those targets which are actually forced to wait and thus gives us a fairer picture of the service process when system utilization is high. In the case where the utilization factor is small, this last measure will be almost exactly the same as the overall system average waiting time due to the fact that many of the targets will arrive when firing is not in progress.

One last note. The reason for not directly considering the length of the idle periods is that in our M/G/1 model it is quite easy to prove that the idle period will follow an exponential distribution with mean length equal to the reciprocal of the target arrival rate.[Ref. 15, p. 208] Unfortunately, we have no firm data on the arrival rate itself but, by using the utilization factor we can avoid this particular difficulty. This will be illustrated in the next section where we shall examine the specific factors and equations that will be used to evaluate the answers to the questions we posed above.

### C. THE SPECIFIC MOES

Before proceeding to enumerate the formulas which will be used to measure our system's capabilities and their implications for actual combat operations, it is important to note that all of the equations that will be used come from the steady-state solutions to the M/G/1 model. This implies that we are looking at a fairly stable combat situation and, as such, it may be necessary to modify our approach if we wish to restrict our analysis to shorter periods, say those of only 30 or 40 minutes duration or to transient situations. Nevertheless, the steady-state approach has a great deal of support for use in our





particular model, especially since we are dealing with artillery systems. Perhaps the most important argument in favor of this approach is to note that US doctrine does not employ artillery in reserve. That is, while direct support artillery units are often given the specific mission of lending primary support to a maneuver unit which may, itself, currently be held in a reserve capacity, the artillery unit is so emplaced on the battlefield that it will be able to fire in support of our units which are still on the FEBA yet it remains prepared to shift its fires or move in support of its primary associated maneuver unit. This particular technique (probably adopted due to the fact that, by usually being employed some distance behind the lines, the artillery units are thereby less vulnerable and, except in very unusual circumstances, do not perform as arduously as the close combat infantry or armor units) thus insures that, regardless of the specific tactical situation, all of our artillery will be available among which we can allocate fire missions. This fact of relatively continuous and constant employment serves to drive our model into functioning near its steady-state capacity.

Moreover, there is another important aspect of artillery employment which also serves to insure a steady-state situation and this is the actual manner of employment. Thus, in addition to not being held in reserve, we find that artillery units are also presented with a wide range of missions which constantly require fires. For example, in an offensive situation the DS units, as well as the others present on the battlefield, are usually engaged in firing preparatory fires prior to an attack in order to block off enemy routes for reinforcements and to sometimes deceive the the enemy regarding our actual objective, then shifting fires onto the objective until our attack forces come within range for their final assault when we then shift our fires again to block enemy routes of retreat and/or counterattack.



On the other hand, in a defensive situation we engage in fires on the enemy's suspected or detected staging areas in an attempt to break up his attack forces as they prepare to move forward against us and, if the attack continues, the artillery is called upon to deliver Final Protective Fires for the main unit under attack as well as to block off any penetration of our lines. In delaying actions the artillery is also rather consistently called upon to fire frequently because its longer range capability means that the weapons can remain in position and take the enemy under fire much earlier in the engagement, before it becomes necessary to continue the withdrawal. Finally, even in the lulls between offensive and defensive operations, the artillery is called upon to fire on targets which our various sensors detect or whose presence is detected or suspected by the various intelligence agencies operating in the combat environment, not to mention the routine harrassing and interdiction fires. Thus, since artillery units are seldom pulled out of the line, it would appear that a steady-state situation is almost certainly present, at least for our initial examination of the model and, having presented these important reasons in support of this view, it is now time to examine the formulas which will be used in our analysis.

## 1. The Server Utilization Factor

Pertaining directly to our first question of interest is the server utilization factor, which is usually represented by the Greek letter rho. However, due to the limitations of my typewriter, I shall use the Arabic letter "r" to denote the utilization factor throughout the remainder of this paper. This term is also known as the traffic intensity for single server systems and is computed thusly:

$$r = \lambda x$$



where  $\lambda$  is the rate of arrival of targets into the firing system and  $x$  is the mean time for a typical target to go through all stages of the firing process, i.e. the time to be completely served. Note that the utilization factor is only defined for values of  $r$  such that:  $0 \leq r < 1$ .

## 2. Residual Target Life

The concept of remaining life or service time until the present task is completed comes directly out of renewal theory where this particular subject is dealt with in great detail. Any reader interested in the full mathematical development is referred to the excellent work by D. R. Cox which is entitled Renewal Theory. [Ref. 6] For our purposes it will suffice merely to state the final results that pertain to our problem. Thus:

$$R_L = \frac{1}{2}(x + \sigma_b^2) = \frac{x}{2}(1 + C_b^2)$$

where  $R_L$  is the residual life of the ongoing mission,  $x$  is the mean service time (as defined previously), and  $\sigma_b^2$  is the variance of the service time distribution (the  $b$  subscript arising from our A/B/m notational system). I have chosen to rewrite the equation after factoring out the  $x$  term in order to put the relationship directly into a form using the square of the coefficient of variation of the service distribution ( $C_b^2$ ) since we shall see this particular term in several other MOEs.

## 3. Length of the Busy Period





While we want to get some indication of the average length of time we will find our units firing, it is also apparent that this number will vary due to the random aspects of target arrival. Therefore, in addition to evaluating the mean value, it may also be helpful to examine the variability in this measure and thus both formulas are presented here:

$$g_1 = \frac{x}{1 - r}$$

where  $g_1$  is the mean length of the busy period and  $x$  and  $r$  are as defined previously and:

$$\sigma_g^2 = \frac{\sigma_b^2 + r(x)^2}{(1 - r)^3} = \frac{x^2 (C_b^2 + r)}{(1 - r)^3}$$

where I have again factored the final form to reveal the presence of the coefficient of variation.

#### 4. Number of Targets Fired on During a Busy Period

Since this measure is directly related to the length of the typical busy period, it is perhaps no surprise to find that

$$h_1 = (1 - r)^{-1} = g_1/x$$

Note, however, that the variance of the number fired is not so easily related to the variance of the busy period, even after some simplification of terms:



$$\sigma_h^2 = \frac{r(1-r) + \lambda^2(x^2)}{(1-r)^3} = \frac{r + r^2 C_b^2}{(1-r)^3}$$

## 5. Average Number of Targets

While the last four measures have concentrated almost exclusively on aspects of the busy period, overall system statistics are still of importance, as was noted in the discussion section above. Thus, here we present the formulas for the average number of targets that we may expect to find in both the overall system:

$$N = r + \frac{r^2(1 + C_b^2)}{2(1-r)}$$

and in the queue:

$$N_q = N - r$$

## 6. Average Firing and Waiting Times

If we examine the average waiting time for all targets, we find the following formulation:

$$W = \frac{xr(1 + C_b^2)}{2(1-r)}$$

and that a simple relationship exists between this number and the average firing time, to wit:



$$T = x + W$$

This simple relationship is what we would expect, even if we had no specific mathematical knowledge of our system, since it merely says that even after waiting, it still takes an average firing period to completely service the target.

## 7. Waiting Time for Delayed Targets

As was pointed out earlier, this measure is of assistance in determining the true delay suffered by those targets which arrive at the firing element when it is already engaged. Thus, we are not surprised to find that the expected wait, given that the target is delayed (EWGD), differs from the overall system waiting time due only to a factor incorporating the probability that we find the system busy:

$$EWGD = \frac{W}{1 - r}$$

And with this last formula we are now ready to proceed with the more mathematical analysis of our system in the following chapter.





## VIII. ANALYSIS

### A. SOME PROBLEMS DUE TO MULTIPLE MOES

In the last chapter we posed seven questions which were considered important in obtaining some insights into problems associated with the timeliness of fires on the battlefield and the implications for standard artillery activities such as maintenance, resupply, etc. In order to answer those questions, a list of some eleven equations was presented, by means of which we could gather numerical information rather than merely debate in the dark about the merits of various alternatives. Unfortunately, eleven different MOEs become rather difficult to evaluate because we thus have a multi-component vector of responses and are confronted with the possible problem of trying to determine which is a better tactical situation in the event that some of our measures show widely differing trends. For example, let us suppose that we are considering only a three component system and that the higher numbers indicate the more favorable responses. Thus we could have something like the following results after comparing two systems:

MOE/System	1	2
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1	5	4
---	---	---

2	5	4
---	---	---

3	5	4
---	---	---



It is quite clear that, in this circumstance, system 1 is the better alternative because it has the more favorable response in each individual component we chose to measure and thus can be said to strictly dominate system 2. On the other hand, had one of the MOEs been equal, say MOE 2, we would still have opted for system 1 because of the dominance in other factors. However, the following is also a possibility that we must consider:

MOE/System	1	2
1	5	4
2	4	5
3	3	3

In this case our basic choice of system is not obvious since we have a very complicated relationship in evidence. Indeed, unless we assign some type of weighting scheme to distinguish the importance of the individual MOEs, we are left only to our intuition to determine which alternative may be the best one to pursue. Clearly, then, the number of such complex possibilities increases tremendously as we increase the number of items that we intend to measure and that, without any specific knowledge beforehand as to the relative importance of either basic trends or of the value of one MOE compared with another, we cannot hope to arrive at a satisfactory method of resolving this difficulty.

As demonstrated above, we can become hopelessly overwhelmed by various relationships between our MOEs unless some definite trends, such as strict dominance, make themselves manifest. Therefore, it would most likely be profitable for us to spend some time to determine whether we can isolate any obvious mathematical relationships between



our proposed MOEs, which would at least reduce the size of the resulting response vector from eleven components down to a more manageable level where important trade-offs might be more visible and could be addressed directly. In this regard, it is important to note that we may expect some series of relationships to be present due to the high recurrence of three common factors in each of our proposed measures, i.e. the server utilization factor ( $r$ ), the square of the coefficient of variation for the service distribution ( $C_b^2$ ), and the mean service time ( $x$ ). At least we would

appear to have some chance for simplification of our problem as contrasted with a situation where our MOEs would not have these various factors in common, so let us see if we can reduce our problem through mathematical relationships prior to looking for trends in output data which may or may not be obvious to us.

## B. SOME MATHEMATICAL SIMPLIFICATIONS

As we noted above, many of our equations are highly dependent on three common factors:  $r$ ,  $C_b^2$ , and  $x$ . Therefore, we may expect that if we examine the simplified equations which were presented in the last chapter, certain regular relationships due to these common elements may emerge and, indeed, such is the case. Four relationships are immediately apparent, since we used them in actually forming the equations in the last chapter. Thus:

$$N_q = N - r$$





$$T = x + W$$

$$EWGD = W(1 - r)^{-1}$$

$$g_1 = xh_1$$

In addition, we also note that N itself is related to W and T in the following manner:

$$N = rT/x = \lambda T$$

which is Little's famous result.[Ref. 18]

Indeed, we note two important aspects related to our simplification attempt. First, we have already managed to reduce the vector of MOE results by almost one half - from eleven components down to six. Secondly, we can readily observe that a rather prominent intermediate factor in our calculations is W, the mean waiting time for a typical target, which is itself determined by  $r$ ,  $C_b^2$ , and  $x$ .

Actually, our dependence on W is further emphasized when we note the relationship between T and W as well as N and  $N_q$ .

Therefore, if we can determine the pattern of behavior exhibited for various values of W based on varying levels of  $r$ ,  $C_b^2$ , and  $x$ , we can relate this directly to four of the

five equations presented above, without the need to measure these other results directly, due to the inherent mathematical dependence we have just observed. Thus we have the six MOEs which remain:



1. The server utilization factor,  $r = \lambda x$
2. The expected waiting time for a typical target:

$$W = \frac{xr(1 + C_b^2)}{2(1 - r)} = \frac{rR_L}{1 - r}$$

3. The mean number served during a busy period:

$$h_1 = (1 - r)^{-1}$$

4. The variance of the number served during a typical busy period:

$$\sigma_h^2 = \frac{r + r^2 C_b^2}{(1 - r)^3}$$

5. The residual life of a target:

$$R_L = \frac{x}{2}(1 + C_b^2)$$

6. The variance of the length of the busy period:

$$\sigma_g^2 = \frac{x^2 (C_b^2 + r)}{(1 - r)^3}$$

Having reduced our list of items of interest due to some rather obvious mathematical relationships involving the expected waiting time for a typical target ( $W$ ), as well as



several of the basic system parameters ( $r, x, C_b^2$ ), perhaps we can continue this process to come up with a still smaller set of MOEs with which to evaluate fire support performance. In this regard, let us take a close look at the residual target life. Here we observe that we still have a strong dependence or relationship with the average target waiting time, to wit:

$$R_L = \frac{(1 - r)W}{r}$$

and thus we could eliminate the residual life from direct examination because of this dependence characteristic. Indeed, we could advance a similar argument in favor of laying aside the average waiting time instead, but I believe that most readers will have a better concept of waiting time than the more abstract notion of the amount of time that remains for completion of service, which is taken from general renewal theory. Thus, confronted with the opportunity to simplify our study by discarding one more MOE from direct consideration, I have chosen to neglect the least familiar term in order to keep the main thrust of the development relatively clear for most readers.

Since we are still looking to reduce our MOE vector, the next item that merits our consideration is the variance of the length of the busy period. We have already ceased any direct consideration of the mean length of the busy period due to its close relationship to the value computed for the average number of targets served during that time period. However, since no obvious relationship appeared between our variance term and our other MOEs, we have continued to include this last item in our list. Nevertheless, is it



really necessary to maintain this item if we are not directly including the mean value to which it is intimately related? I would propose that the answer to this question should be in the negative, and that we also drop this term from our main list of MOEs, at least until it becomes necessary to do an in-depth analysis of the busy period which cannot be handled by simple extension of our results from other areas.

Finally, in our continuing quest for simplification, let us examine the server utilization factor ( $r$ ). While this certainly has an effectiveness aspect associated with it, in that it is by means of this parameter that we can estimate the percent of time that we are in a firing status, with all of the concomitant implications which then pertain to our fire support system, nevertheless,  $r$  is exclusively a function of both the mean service time ( $x$ ) and the arrival rate of detected targets into our system ( $\lambda$ ). Unfortunately, while we can hope to measure or evaluate the purely mechanical aspects of fire support, which also provides us with estimates of the mean service time, as well as more general human response data, even without checking various distributional assumptions, we have no such information regarding the target arrival rate, since it is highly dependent on the particular scenario we are observing, which includes such important factors as enemy tactics and dispositions, etc. Indeed, this is one of the reasons we assumed the long-run Poisson character of the arrival process, because we have no concrete information. Yet, the server utilization factor is extremely important for evaluating all of our other formulas and we would thus appear to have an insoluble dilemma on our hands. However, we do have a reasonable way out of this impasse.

For any given artillery weapon, or system, we most certainly have what amounts to an average service time per





target when we consider the amount of time needed to destroy all possible types of enemy targets averaged over the various firing disciplines which our tactics call for. Thus, we can consider that  $x$  is probably known, or can be computed, for each tactical situation, or process, based on whatever firing discipline we wish to employ. Then, with the mean service time fixed for a given weapon or system, if we vary the rate of target arrivals, this is equivalent to varying  $r$  and vice versa. Thus, if we vary  $r$  in our study, we can observe how our system responds as a result of different tactical situations, which are thereby represented by the differing arrival rates that would be apparent could we actually test each situation. Also, by looking directly at the effects of different utilization values, we can determine whether any patterns are prevalent at a particular level of activity, which may motivate further experimentation or study in that particular area of combat interactions. However, if we use this approach, then we must remove the server utilization factor from our list of MOEs, since it now becomes a surrogate variable for the arrival rate, leaving us with the following three-component vector: the average waiting time for a typical target, the average number of targets served during a typical busy period, and the variance of the number of targets served per busy period. Since we can compute the values of any of our other items of interest (in our original list of MOEs) from these three measures, they alone are sufficient, for the remainder of our analysis task, for providing information about the operations of the fire support system.

Two brief observations are in order before proceeding with our study. First, due to our lack of knowledge about the exact arrival rate intensity, it is now clear why our initial formulas were modified so as to eliminate any dependence on  $r$  and to put the final form completely in terms of the server utilization factor and the square of the



coefficient of variation for the service distribution. Second, rather than require actual values for our various service times, which will be different for each type of tactical firing discipline as well as for each weapons system we may wish to consider, let us factor our formula for the average waiting time thusly:

$$W_F = \frac{W}{x} = \frac{r(1 + \frac{C^2}{b})}{2(1 - r)}$$

where  $W_F$  will be the factor affecting the increase in service time independent of any particular average service procedure, being solely dependent on the actual system utilization and the variability of the actual service distribution. With this multiplication factor, we can then adjust any actual or proposed service time to account for the effects of congestion and randomness in both the arrival and service distributions. Thus, we will also have a pure number to use for comparisons when we examine the effects of different distributional assumptions.

## C. STUDY RESULTS

In the last section we managed to reduce the scope of our analysis down to the examination of only three specific measures: the average waiting time for a typical target, factored to show the distributional effects; the average number of targets served per busy period; and the variance of the number of targets served in a busy period. Let us now proceed to examine each of these measures in detail.

### 1. The Waiting Factor



As we noted in the previous chapter, the length of time that a typical target must wait until it can be fired upon has a direct bearing on our ability to provide adequate fire support to friendly forces. This becomes increasingly vital as the mobility of our target increases, since that factor, in itself, raises the possibility that if the waiting period is long enough, the target may become unavailable for fire. Thus, it could be claimed that the artillery system failed to respond adequately in that tactical situation. In addition, the concept of the "dedicated" battery places an even greater emphasis on the timeliness of fires than was necessary prior to the adoption of this particular tactical procedure.

Having reminded ourselves of the importance of this specific measure, let us examine the implications different service distributions have on our system's behavior. Fortunately, as explained in the last section, we can factor the specific average service time out of our equation and thereby examine a "waiting factor" which is a pure number not dependent on mean values alone. Thus, our average waiting time per target will be a simple relationship of the form:

$$W = xW_P$$

where we need merely take the average service time of our system and multiply it by the waiting factor due to our specific distribution and server utilization factor. Since the mean service time will be invariant with respect to the distributions we use, i.e. the major differences between distributions will be in other areas such as variance, skewness, etc., we need only examine the waiting factor itself in order to understand the net effects various





distributions will have on our average waiting time. The formula is therefore reproduced here for reference to the analysis in this section:

$$W_F = \frac{r(1 + \frac{C_b^2}{2})}{2(1 - r)} = \frac{r}{(1 - r)} \cdot \frac{(1 + \frac{C_b^2}{2})}{2}$$

If we examine the factored form on the far right of the above equation for the waiting factor, we observe that, with respect to  $r$ , the server utilization factor, we have a family of hyperbolic curves of the form  $y = ax(1 - x)^{-1}$ , which has asymptotes at  $r = 1.0$  and  $W_F = -1.0$ , as in Fig 6.

However, due to the limitations we put on the server utilization factor earlier, namely, since it represents the percentage of time the battery is engaged in firing tasks,  $r$  must lie between the values of zero and one, we are only interested in the behavior of this family of curves in the interval labeled "A". This satisfies our intuitive feelings about the system, in that we can never have a negative average waiting time, since that is a physical impossibility. The best we could do is have arrivals which occur just as the server becomes free and thus have no waiting period at all. Indeed, our "a" parameter can never be negative, since the only variable in its determination is the square of the coefficient of variation of our service distribution  $(\frac{C_b^2}{2})$  which is always positive. We should also

note that the behavior of the waiting factor in this restricted region satisfies our intuition in another important respect. That is, as the traffic intensity of our system increases, longer factors for the waiting period are



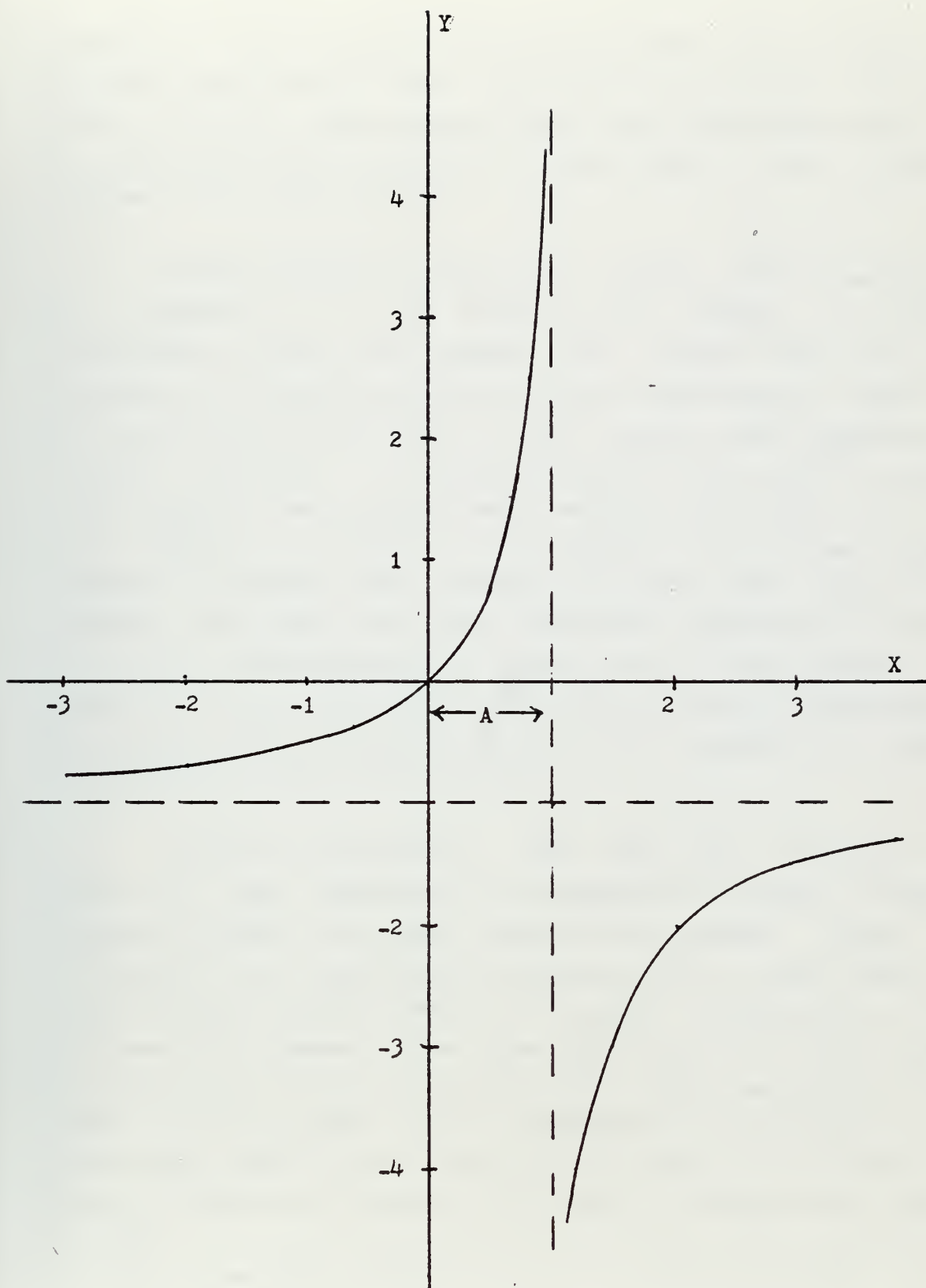


Figure 6 - THE HYPERBOLIC FAMILY



produced, which is just as we would expect even without any detailed knowledge of the applicable equations. As the server becomes increasingly busy, newer arrivals must wait for their turn and this drives up the value of the average waiting time in the system.

Having found the family of curves which describes the behavior of our system with respect to server utilization, we can now examine the effects due to the different distributions played by our simulations. Since we have previously noted that an M/M/1 system would represent a long-run tendency of random behavior, and an M/D/1 system would describe our deterministic models, let's look at how they appear as we focus on the interval "A". The next three figures illustrate the effects observed for these two systems. The reason for three different figures is in order to show the differences more clearly due to the changes in scaling on the vertical axis as the value of  $r$  increases. Each graph illustrates the effect of the increase in server utilization on the waiting factor.

It may be noted that, for the M/D/1 system, the square of the coefficient of variation is equal to zero, thereby yielding the lowest curve on the graphs. Thus, as we expected, the deterministic system gives a lower bound on random behavior since there is no random contribution from the service element. On the other hand, the M/M/1 line, with the square of the coefficient of variation equal to one (since the variance of an exponential distribution of inter-event times is the square of the mean), is merely a form of asymptote revealing the long-run tendencies. While we expect that most processes we observe on the battlefield will fall somewhere between these two limiting curves, there are distributions (e.g. the Cauchy) which have such large variances as to compel us not to ignore the possibilities of curves even higher on the graphs (even though this is



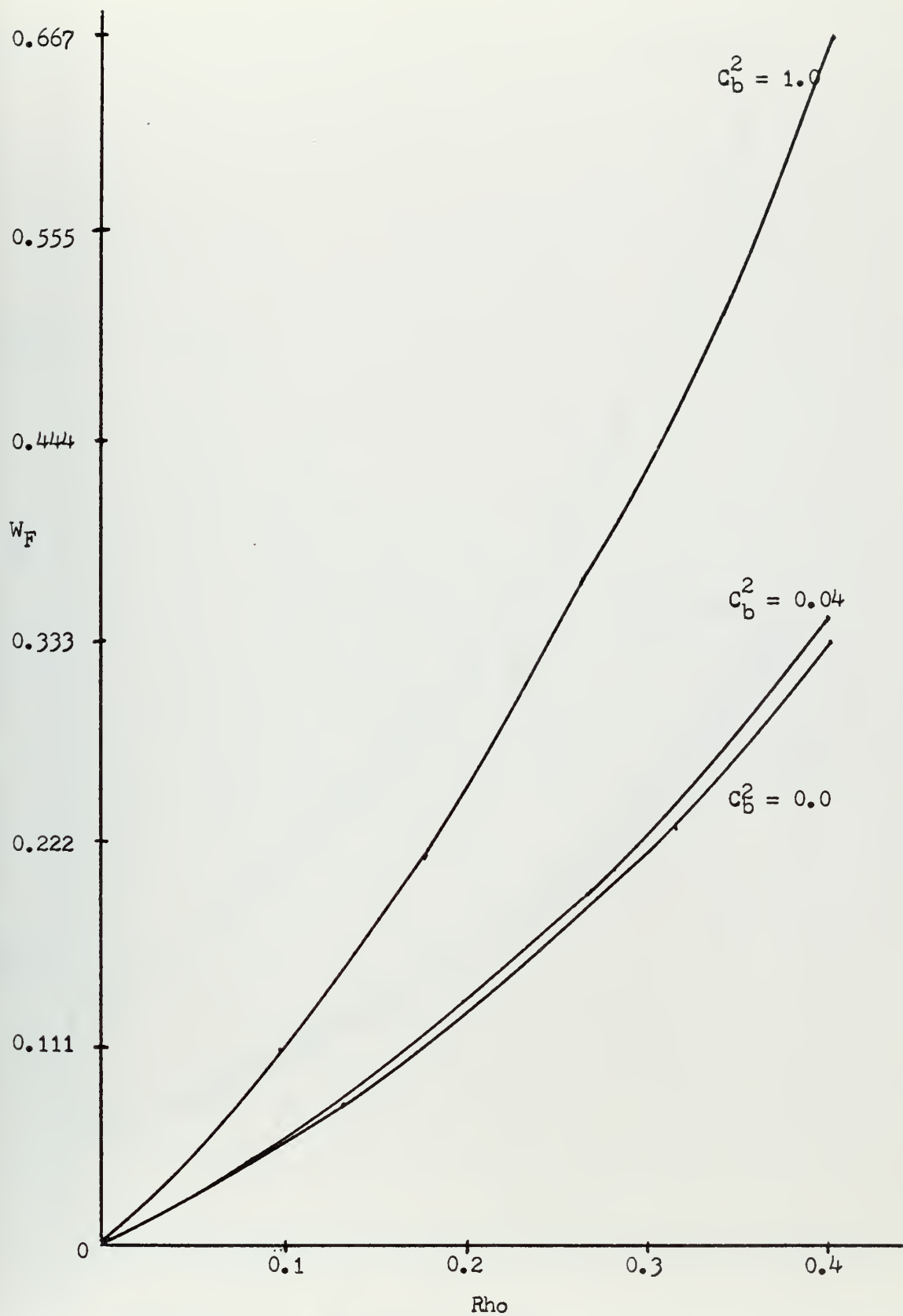


Figure 7 - WAITING FACTOR VS SERVER UTILIZATION FOR  
CONSTANT LEVELS OF RANDOMNESS-I





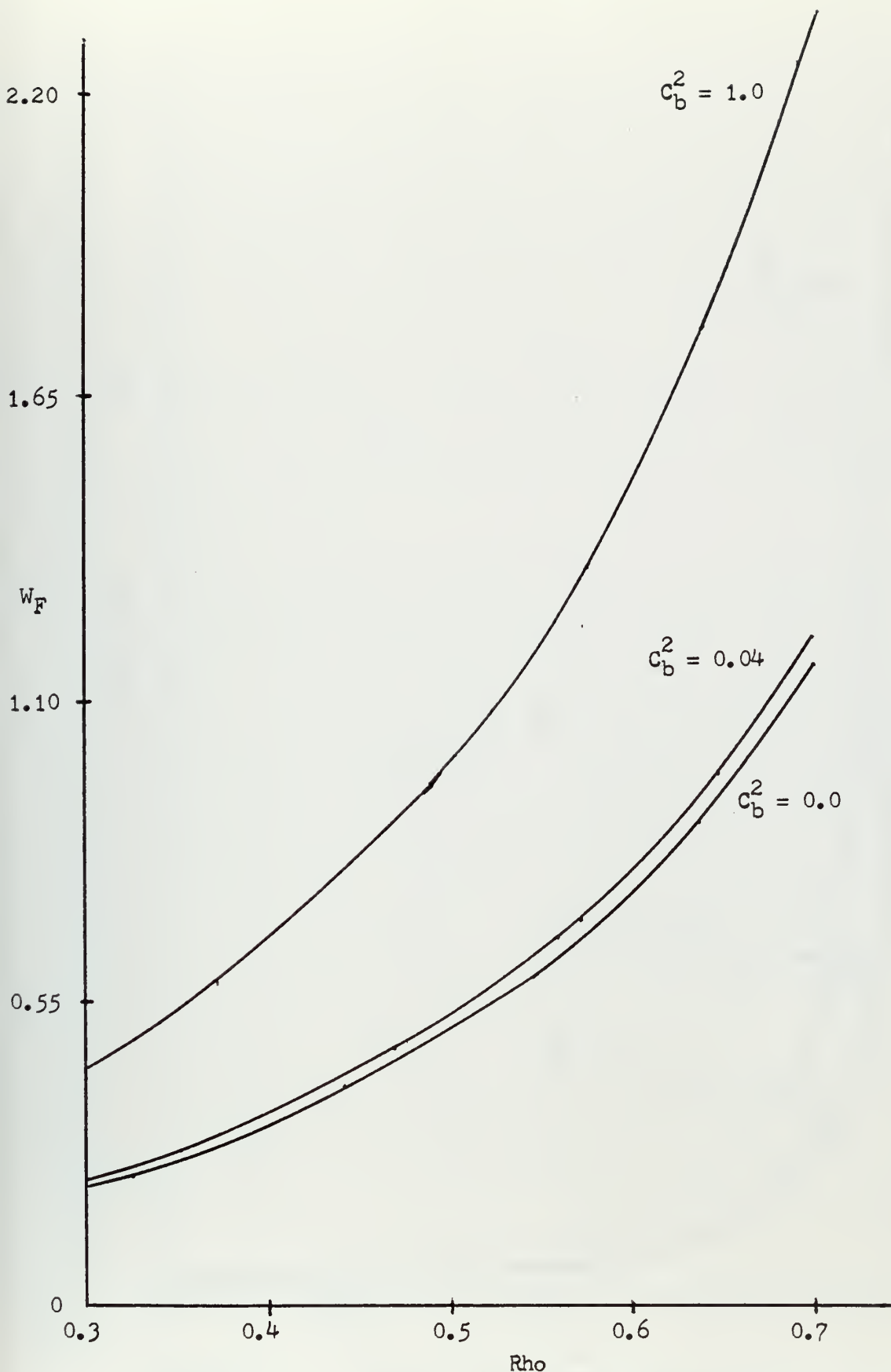


Figure 8 - WAITING FACTOR VS SERVER UTILIZATION FOR  
CONSTANT LEVELS OF RANDOMNESS-II



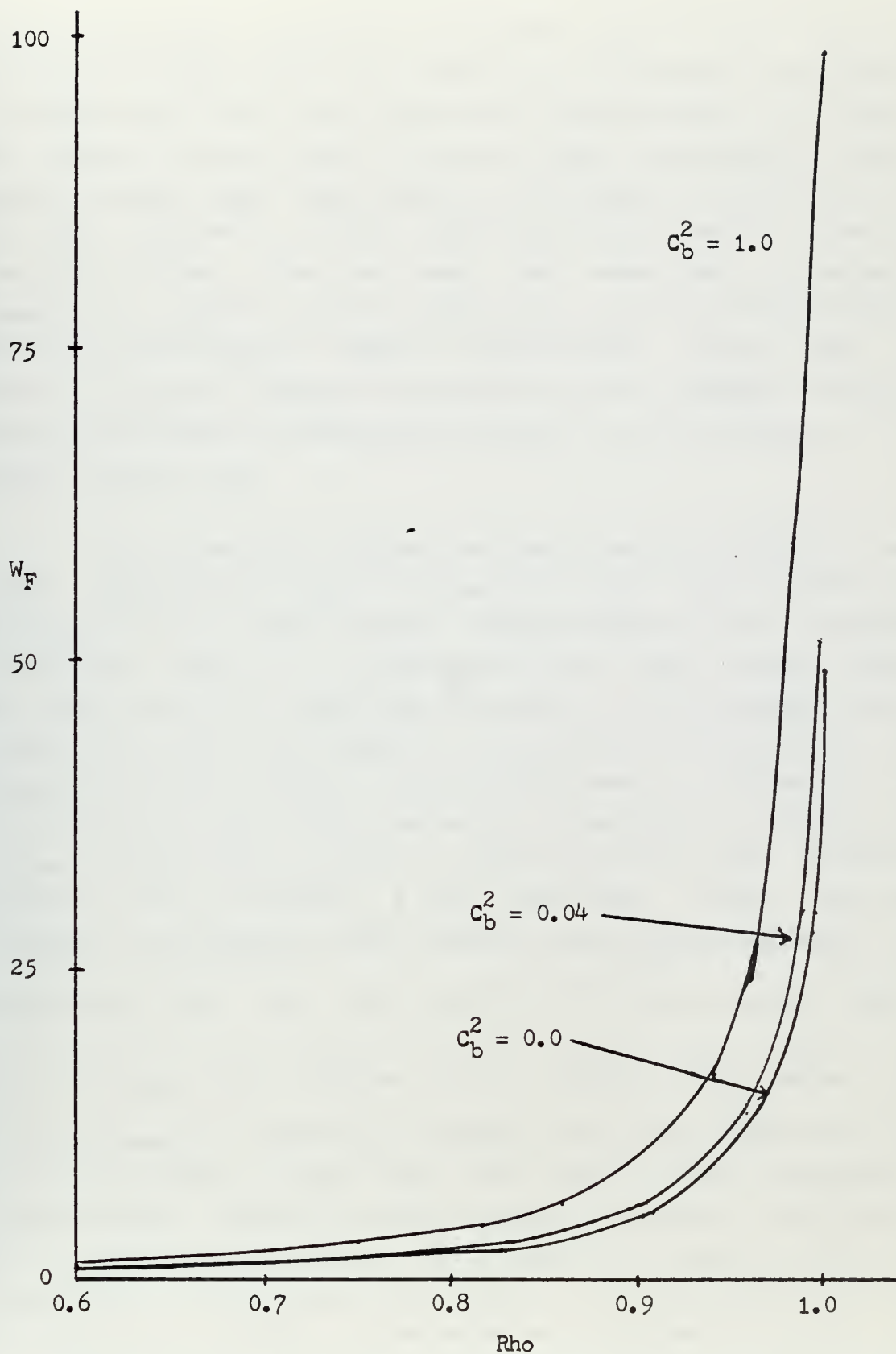


Figure 9 - WAITING FACTOR VS SERVER UTILIZATION FOR  
CONSTANT LEVELS OF RANDOMNESS-III



highly unlikely). However, we would expect that, if our stochastic simulations are accurately portraying randomness, we should find that their choice of distributions will result in members of our family of curves which approach the M/M/1 case. On the other hand, should the choice of distributions in the stochastic models yield long-term results close to the M/D/1 line, then we must ask some very important questions regarding their usefulness over the deterministic models in portraying combat interactions. With this in mind, we can now examine the effects of the DYN TACS and CLGP COFA assumptions by determining where, in our diagrams, the final results lie.

In an earlier chapter we presented the DYN TACS and CLGP assumptions which reflected that these models and studies chose to use Normal distributions with standard deviations equal to 20 percent of the mean response times for each stage of artillery service. By imposing this restriction on the variance of the Normal distribution, we observe that the square of the coefficient of variation is then equal to 0.04 under the model assumptions. However, it turns out that this is only an upper bound for the system's behavior (see Appendix B for details). Thus, we can evaluate the effects of the DYN TACS and CLGP assumptions by observing where the line for  $C_b^2 = 0.04$  lies on our graph.

In so doing, we observe that there is almost no distinguishable difference between the final results due to the deterministic model and our use of the stochastic simulations. However, before leaping to such a conclusion, let us look at the values of the waiting factor from the standpoint of varying the square of the coefficient of variation and looking at the effects at different levels of server utilization. As we do this, it will be noted that the waiting factor is a linear function of the square of





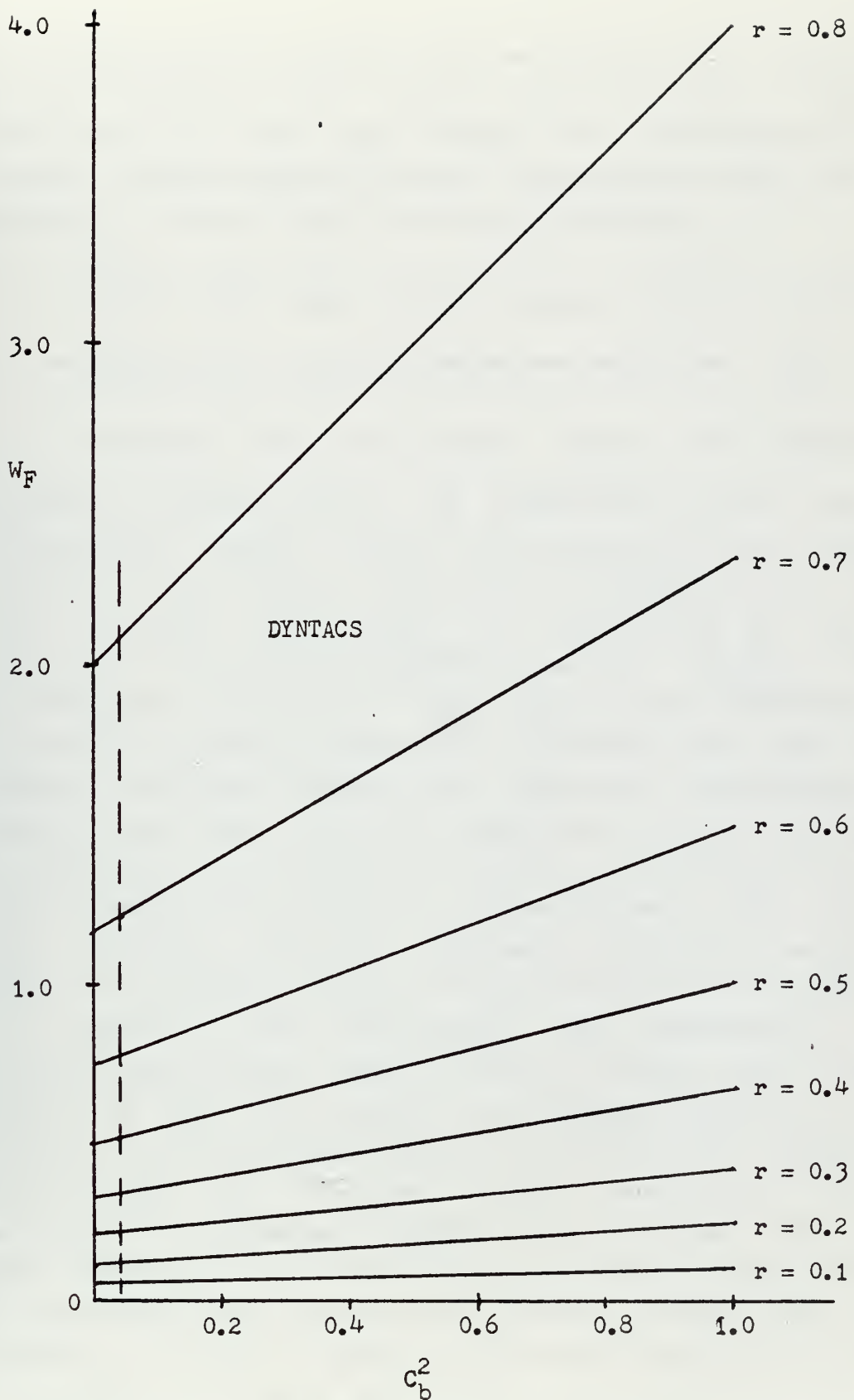


Figure 10 - WAITING FACTOR VS RANDOMNESS PLAYED FOR CONSTANT SERVER UTILIZATION



the coefficient of variation and we can see the effects on waiting somewhat more clearly since we are, in essence, concentrating on the gap between our deterministic and long-run curves when we isolate a particular value of  $r$  and examine the effects due to server variability.

With the value of the waiting factor remaining on the vertical axis, and our randomness parameter  $(C_b^2)$  along the horizontal, we can readily observe the effects of different distributions. We immediately note that the deterministic models yield the intercepts on the vertical axis and I have chosen to cut the line segment at the  $M/M/1$  point, thereby emphasizing our examination of the gap between the two original bounds we set up. However, should we discover later that our square of the coefficient of variation is greater than one in practice, we can simply extend the line segments as required. Nevertheless, by isolating the gap, we find ourselves in the desirable circumstance of measuring effects and trends which, like basic probabilities, fall on a scale between zero and one very naturally. This is a fortunate occurrence, especially since the parameter which is varying along the interval is our measure of the variability (or randomness) of the service distribution. Thus, unless we obtain valid evidence indicating that we should not confine ourselves to this interval on an exclusive basis, we may view our results as illustrating the waiting factor associated with the percentage of randomness inherent in our system. Thus, zero will indicate no variability at all (the deterministic case by definition) and the value of one will signify full random behavior, in this case our long-run tendency. Now, using this point of view, we observe that the DYN-TACS and CLGP assumptions, having their upper bound of 0.04, or only 4 percent randomness, are thereby producing final simulation



and study results that are statistically indistinguishable from the deterministic models. Indeed, when we consider that the great percentage of hypothesis tests use a 90 or 95 percent confidence requirement, were we to test our DYN-TACS results in comparison with the deterministic models of the same process, we would be forced to conclude that no significant differences exist.

Based on these results, we are now forced to confront the question posed just a few paragraphs ago. Namely, if we cannot tell the difference between the outcomes of most stochastic models from those of the deterministic ones, which should we use in modeling military problems? The answer is, of course, quite clear. We should choose the least costly technique in making our various studies, since the results do not justify special concentration on stochastic models, which are usually the more expensive. However, this result thus far pertains to only one of our final three MOEs and, even going back to our original list of seven questions in the last chapter, we note that only four elements are directly affected by this development: the residual target life; the average number of targets in the queue and in the system; the expected service and waiting times; and, finally, the expected wait given that we are, in fact, delayed. While all of these are important aspects of the fire support system, we have yet to consider our other two MOEs, on which the rest of our system depends. Therefore, it would be inappropriate to continue with the arguments pro and con stochastic models until our trends are either reinforced or annihilated by later developments. In view of this fact, we will defer further discussion of this point until after the results of our analysis of the other two factors has been completed and we will then be in a position to determine whether this indistinguishability of results is still important and applicable to the entire system, or merely a property of our





first MOE.

## 2. The Average Number Served per Busy Period

The second of our major MOEs chosen earlier is the average number of targets served per typical busy period:

$$h_1 = (1 - r)^{-1}$$

and we immediately note that the variability of neither our arrival nor our service process affects this value. Again, this is an intuitively satisfying result when we consider that the average number of targets served during a busy period should only be affected by the average number which arrive and the average service time, all of which is incorporated in the server utilization factor ( $r$ ). Consequently, regardless of the service distribution chosen, this average will be constant for a given traffic intensity and we can examine the variation due to different utilization factors from the graph in Fig 11.

This result has an interesting implication for our deferred comparison of the stochastic and deterministic models, in that it provides us with some significant insights into a paper deleivered at the 13th U.S. Army Operations Research Symposium (AORS) by personnel from Rodman labs.[Ref. 4] In that presentation, it was noted that a comparison of the average of 10 DYN TACS runs with one run of a deterministic model (in this instance the FAST model) simulating the same general battlefield activity, i.e. company and battalion level actions, produced results in casualty figures and casualty ratios which were statistically indistinguishable at standard confidence levels. Indeed, the actual casualties for the Red and Blue





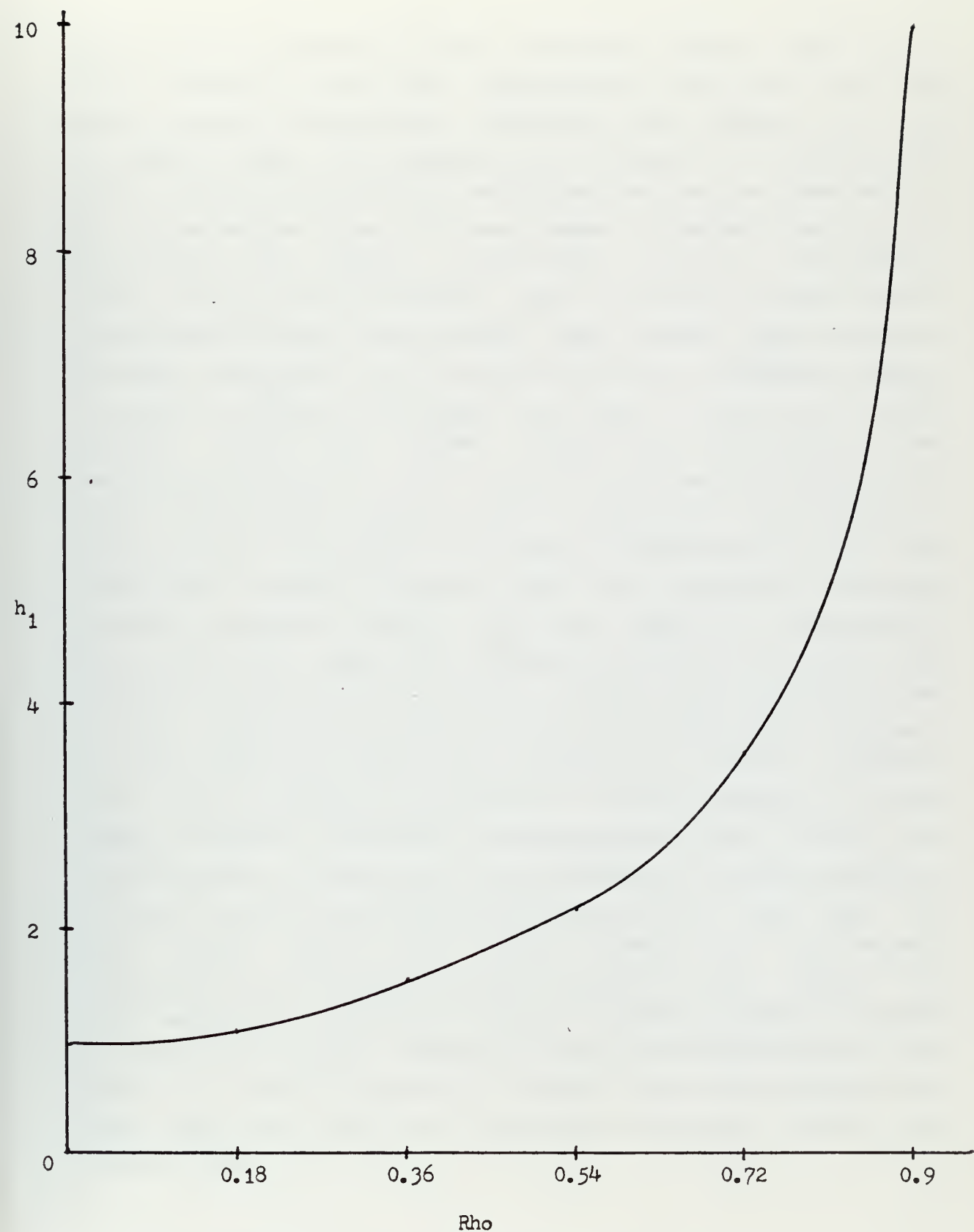


Figure 11 - AVERAGE NUMBER SERVED PER BUSY PERIOD



forces were so closely matched between models (less than 0.36 standard deviations difference for Red and less than 0.58 standard deviations difference for Blue) as to seriously raise the question, even then, as to whether the least costly deterministic techniques are to be preferred. Our observation about the mean number of targets served per busy period thus assumes a significant role in explaining these earlier study results. That is, when we observe that casualties can only occur during busy periods, then, when we average casualties over various runs of a stochastic model we are, in essence, averaging the number served per busy period. Since this number is the same regardless of the service distributions used, we would be greatly surprised if major differences occurred when we compared a deterministic model and the mean values of several stochastic runs for the same battle process. Indeed, since both models use the same casualty producing weapons, the mean number of casualties per ammunition type will be almost identical, and thus any variation we observe in the final output must be solely due to the variability inherent in our distributional choices. The most important point here, however, is that our present MOE, while seemingly unrelated to the variability in our models, further reinforces the importance of our choice of distributions, since only the variance of the distributions is going to cause any different results and, thus far, we have noted that the current choice of the Normal distribution with small standard deviation with respect to the mean, has produced results indistinguishable from deterministic models. However, since the variability of our models is still of primary concern, let us now examine the final MOE, the variance of the number of targets served per busy period.

### 3. Variance of the Number of Targets Served per Busy Period



We now come to the final component of our simplified MOE vector: the variance of the number of targets served per busy period. Up to this point, our previous considerations of the first and second components has reinforced the general results of our study, rather than revealing any different tendencies inherent in actual operations. Therefore, we would expect that we are likely to see even further confirmation regarding the lack of variability in our distributional choices since this current MOE is also dependent on the relationship between the server utilization factor and the square of the coefficient of variation as reproduced below:

$$\sigma_h^2 = \frac{r + r^2 C_b^2}{(1 - r)^3}$$

While we are now concerned with a family of cubic equations in terms of  $r$ , we are still concentrating on the interval for  $0 \leq r < 1$ . Looking at the details of this scheme as we vary  $r$  for different fixed values of the square of the coefficient of variation of the service distribution, especially concentrating on our two limiting lines and the DYN-TACS case, we observe the same general phenomenon at work as noted in considering the waiting factor. Indeed, the similarity of the results is so remarkable that, were we not aware of the actual differences in the equations describing these MOEs, we would almost be inclined to believe that we are looking at the same graphs merely relocated in the paper. Indeed, except for scaling changes, the behavior of each of these MOEs (the first and third) are almost identical. Thus, if we again concentrate on varying the randomness parameter and holding  $r$  fixed, we observe that the relationship is once again linear, but with a change in both slope and intercept. And, since all of our previously





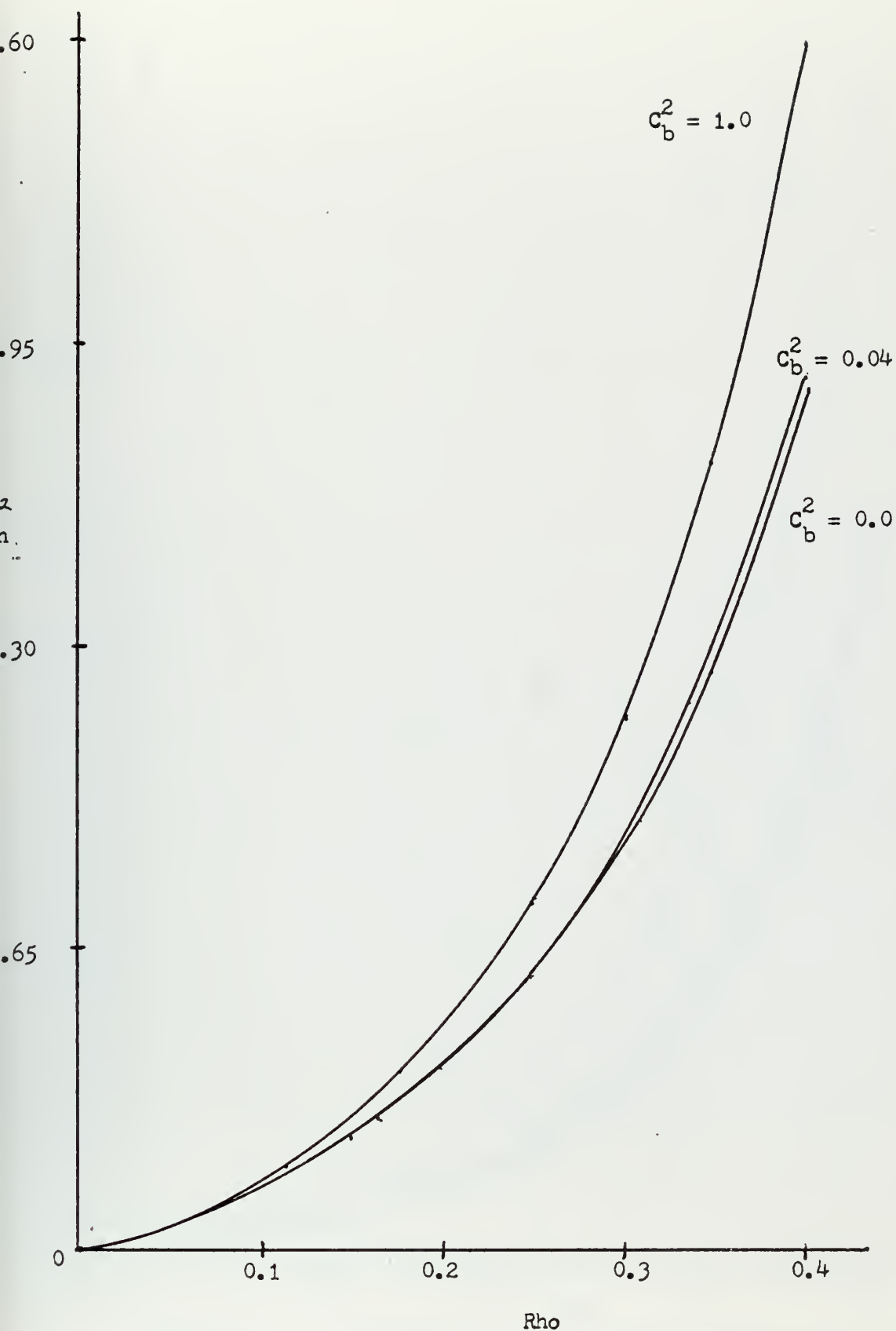


Figure 12 - VARIANCE OF THE NUMBER SERVED VS UTILIZATION FACTOR FOR CONSTANT LEVELS OF RANDOMNESS-I



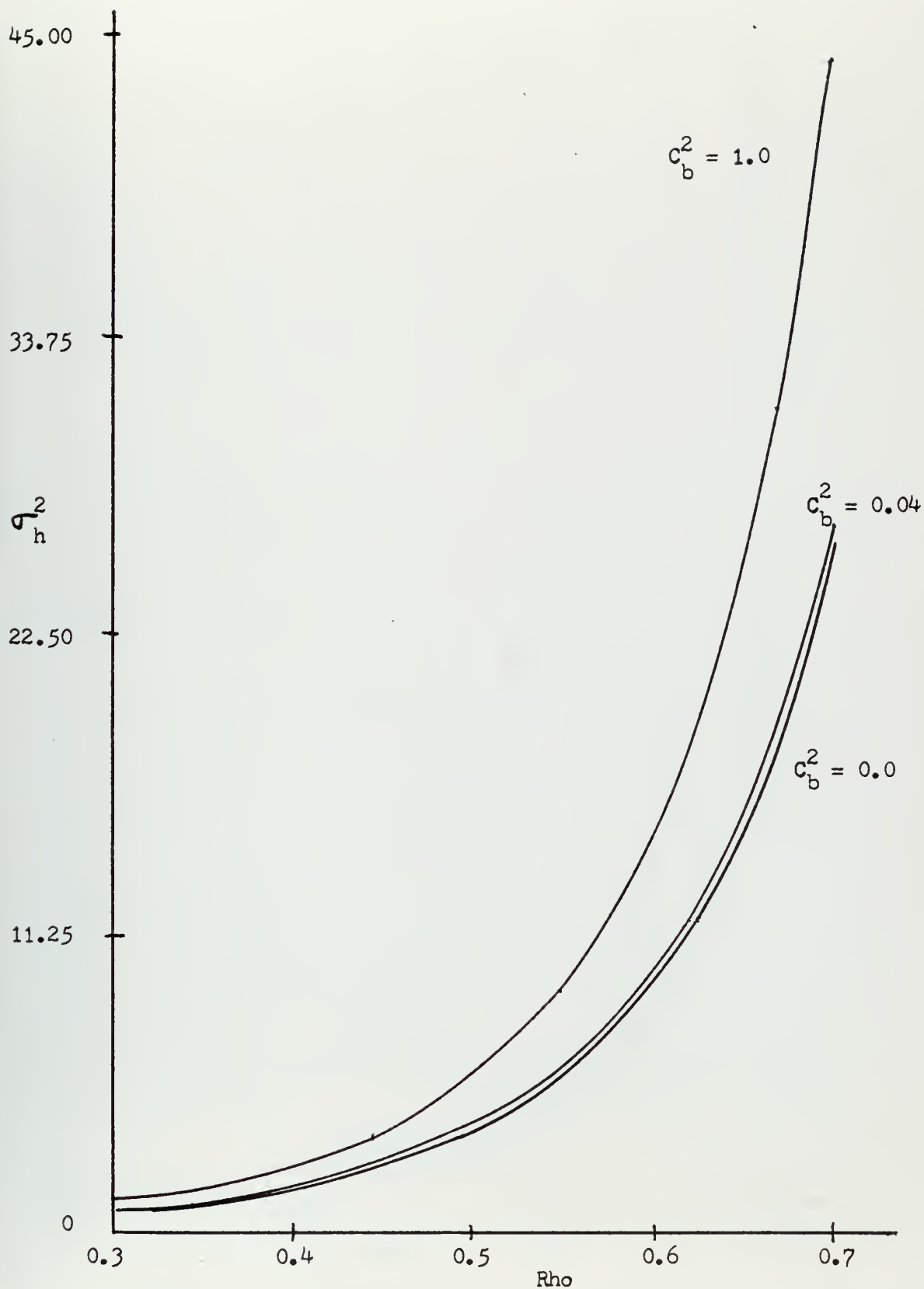


Figure 13 - VARIANCE OF THE NUMBER SERVED VS UTILIZATION FACTOR FOR CONSTANT LEVELS OF RANDOMNESS-II



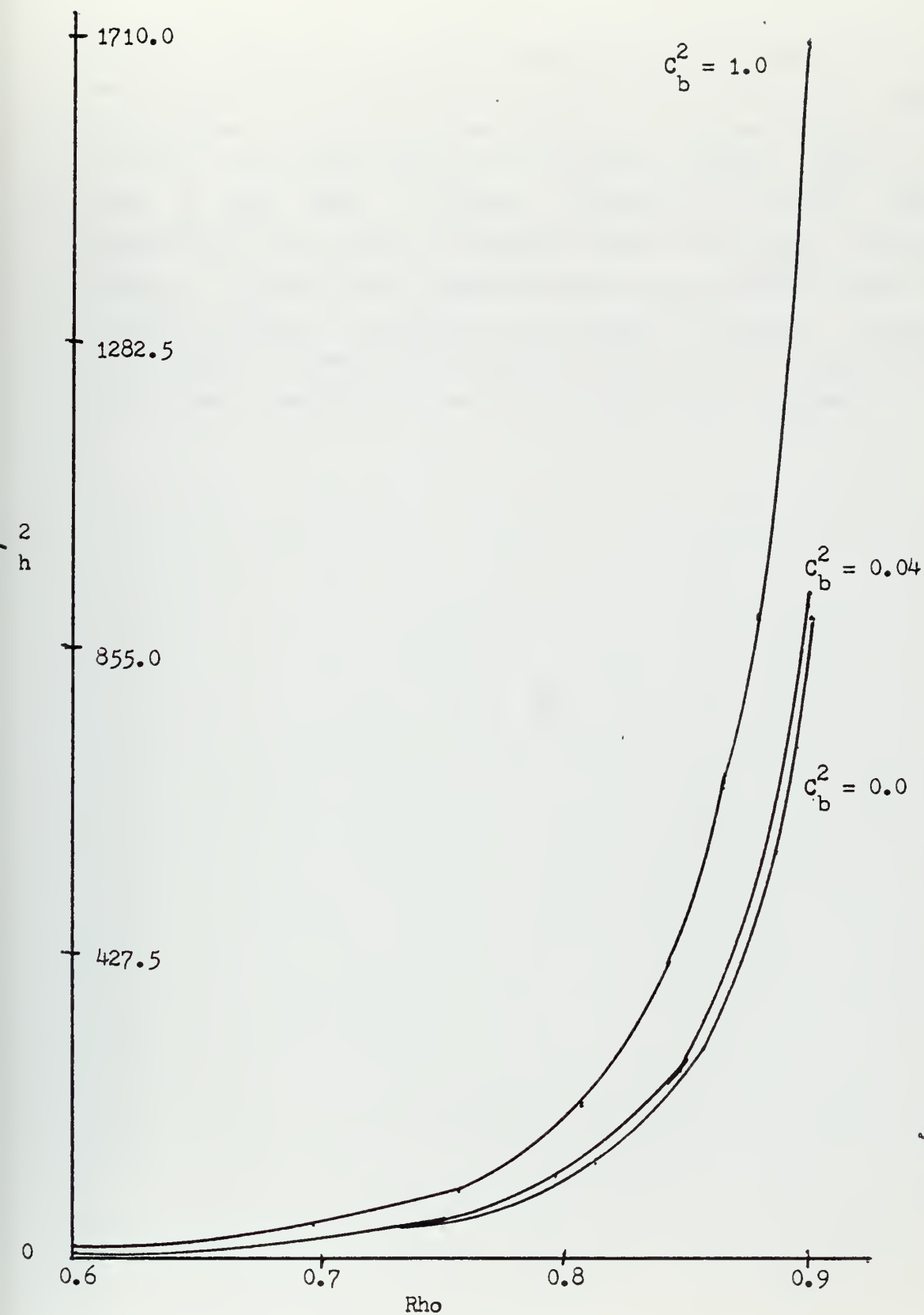


Figure 14 - VARIANCE OF THE NUMBER SERVED VS UTILIZATION FACTOR FOR CONSTANT LEVELS OF RANDOMNESS-III



noted trends are uncontradicted in this last MOE, we may safely say that, since our earliest list of seven questions simplified to this reduced form, all of our conclusions and results are applicable to the entire system. Finally, with all of our MOEs yielding consistent results from the analysis, it is time to consider what our final conclusions should be, and what implications they have for fire support tactics and the analysis of combat models. Since this concludes the analysis portion of the paper, I shall address these new considerations in the following chapter.





## IX. CONCLUSIONS AND RECOMMENDATIONS

### A. BASIC CONCLUSIONS AND THEIR IMPLICATIONS

In the last chapter we saw how our three basic MOEs, upon which the answers to the seven basic questions in the MOE chapter essentially depend, are influenced by the server utilization factor and the variability inherent in our choice of service distributions. Since the relationships between the waiting factor and the variance of the number of targets served per busy period, with respect to the randomness parameter (the square of the coefficient of variation of the service distribution) are linear in nature on the interval between zero and one, it is possible to reasonably address the problem from the standpoint of looking at the percentage of variability played by our models. Clearly, the deterministic models, by definition, play zero percent randomness and, on this scale, the long-run random model (M/M/1) represents 100 percent, or maximum, variability.

Inasmuch as the major argument in favor of stochastic simulations stresses the importance of their being able to portray the random elements of combat with high fidelity, it is rather shocking to observe that the specific assumptions used in many stochastic models, such as DYN-TACS, yield results which are statistically indistinguishable from those gained from deterministic models. Indeed, using the percent randomness approach, we have seen that DYN-TACS underplays long-run randomness by a factor of 96 percent. While this



tendency was hinted at in the Rodman labs paper previously cited, our queueing approach has given a formal proof that what was observed there was not an anomaly. By the very fact of choosing Normal distributions with relatively small standard deviations, we are forcing our stochastic models not to play what may be the true random nature of combat interactions. Of course, the question which arises from this conclusion is: Do we really believe that combat has significant random features and, if so, how do we adjust our models to reflect this reality? On the other hand, it may be that the primary random influences on the combat process do not appear in regards to weapon and human response times, but affect other aspects of the battlefield process instead. However, before addressing this possibility, let's consider what the effects of our current modeling techniques and assumptions are, in the event that true combat is more random than is currently being simulated.

In the process of simplifying our original list of MOEs in the last chapter, we noted the high degree of dependence for all of our measures on the average waiting time for a typical target and that this could be traced to a waiting factor independent of the mean service time. If we look back at our graphs of the hyperbolic forms, it is readily apparent that the greater the amount of randomness we portray in our models, the greater will be this adjusting waiting factor, since our deterministic case represents a lower bound. The net effect, then, of not fully playing whay may be true randomness of the combat process, is to underestimate the amount of time that targets are delayed and also the time they require for adequate servcie. This has serious implicatons regarding the level of casualties inflicted by our forces and on the behavior of the model itself. Thus, if targets are not delayed in service as they should be, due to the actual randomness we want to portray, this means that we are firing at them and producing



casualties faster than we should. This has a great effect on our overall system because, once a target is served, we then begin firing at the next one sooner than is appropriate and the effect "snow balls". The final result is that the targets we have fired upon are being killed earlier in the battle and faster than would normally occur and, moreover, we are killing more targets than is justified, thereby overestimating the effects of our weapons. This is a very serious problem since, by overestimating casualties, we are providing false effectiveness scores for most of our COEAs, since the majority of them base weapon effectiveness on the number of casualties produced. This situation is further aggravated by the fact that weapons effects, themselves, are definitely non-linear. That is, if we double the number of artillery tubes firing, we do not necessarily double the number of casualties produced. Thus, we run the risk of choosing the wrong weapons systems for development if we are erroneously estimating performance in the tactical environment portrayed in our models. In addition, by the non-linearity of the weapons effects, we have no guarantee that the overestimation is the same for different type weapons, we merely know that it is present.

In addition to overestimating casualties, we are also likely to be overestimating the number of rounds fired. This results from the fact that, in a given time period, we are actually engaging more targets than we should, even though the ones we do engage are killed in the same manner. That is, while it will still take the same number of rounds to kill an individual target, regardless of the randomness in response times (since the drawing of the random results in the casualty subroutines is independent of the arrival and service process), the manner of casualty production is not affected but the number of targets engaged is affected. Consequently, the more targets engaged, the more rounds that are fired, thus leading to an overestimation of ammunition





expenditures. Since ammunition expenditures are usually included in the costing equations of COEAs, this also erroneously affects the cost of the weapons systems we are comparing, in addition to the errors introduced on the effectiveness side, which can then lead to the choice of a weapon not appropriate for our needs due to our misvaluation of the combat process.

While these implications are extremely important for military analysts and the resulting doctrinal changes that are incurred due to the adoption of different weapons systems, the entire result hinges on whether or not the combat process is close to the long-run random outcome rather than being more deterministic, as we are currently modeling it even in our stochastic simulations. Certainly there appears to be a great deal of historical evidence which indicates that the side with the greater force and/or better weapons does not always win battles, thus reinforcing our belief that there are significant random features that affect the outcome of combat. The main question is: are these features operating in the area of human and weapon response times, or do they make their impact on combat in some other manner which we have not yet modeled? If the answer to this question is that the random effects we do observe are the results of regular human and weapon responses, then we are not currently modeling this facet accurately in our choice of distributions, and more experimentation and validation of our distributional choices are required. On the other hand, even after some basic validation, it may occur that our current distributional choices are, in fact, the appropriate ones to make. In this case, we are in a very desirable position as regards choosing models to evaluate weapons effectiveness, for we can then elect to use whichever technique is the least costly in terms of our constraints on time, manpower, computer needs, and the general budget. Since most



stochastic models are usually more costly than many of the deterministic ones, it would appear that for statistically equal effectiveness, we would do much better to choose the least cost method, even if we are overestimating the various elements noted above, since we would still be making this erroneous estimate but at least we can then use the savings from the changes in our modeling methods in more profitable ways. However, before we can safely choose this alternative, we must be certain that we really belong near the deterministic outcomes when looking at our stochastic models. Unfortunately, this means that we must verify our distributional choices, or else we could merely be perpetuating our current errors at less cost. Now, this is the point where most study papers and theses usually stop. That is, the author calls for further investigation and closes his paper. However, rather than leave the reader with the idea that such validation might be difficult and expensive to perform, I intend to present, in the next section, a way to verify the distributional choices of our models which can be implemented at essentially no cost to the U.S. government.

#### B. A NO-COST SOLUTION TO THE VALIDATION PROBLEM

Having demonstrated the importance of the distributional assumptions used in our stochastic models, as well as some of the important implications current choices have on military analyses, it is crucial to determine just what the true distributions describing human and weapon responses actually are. Here, is where I believe that military Operations Research personnel can probably make their best contribution to the discipline. Since most combat models are products of civilian "think tanks" or private contractors, it is highly unlikely that significantly new



contributions will occur in this area as a direct result of the employment of on-duty military personnel. Likewise, with the daily requirements for numerous studies and analyses, the working climate, with its pressures to meet deadlines, is not really conducive to the academic aspects of expanding the frontiers of the discipline through concentration on theory to the exclusion of practice on the part of military analysts. Rather, it is on the operational level that the trained military analyst may be likely to provide the insights not available to the theoreticians or the civilian modelers. A specific case in point is this very study.

Having examined the fire support system as it currently is implemented in daily operations, and some of the relevant changes brought about in response to the 1973 Mideast war, as well as current modeling procedures, it would appear that, even with our simplified queueing formulation, a great deal of extra work is required to validate our present distributional assumptions. However, when we consider the full gamut of artillery operations on a regular basis, we find that the data we need for this validation step is already being gathered but, due to the lack of communication between the researchers and the daily operators, this vital information is discarded, rather than sent to the analysis groups which need it. This unfortunate circumstance has arisen precisely because the operational personnel, untrained in analysis techniques and bewildered by the vast complexity of military models, are not aware that the data is needed. Likewise, the modelers, being far removed from either the military itself or lower level operations, are not aware that the data is already being collected. Therefore, let's look again at the daily operational activities of fire support units to see just what data we can gather in regards to verifying our modeling assumptions.





As we examine current operations in an effort to find out where this needed data is collected, we observe that every artillery unit which has a combat contingency mission (rather than a training task in the event of war or mobilization) is required to take an evaluation test at least once a year to determine if the unit is "combat ready." While this testing procedure is known by different acronyms, such as ATT, ORTT, etc., the same basic evaluation procedures are followed. That is, the unit undergoing the testing receives a mission-type order regarding a combat situation and, after the start of the test period, all operations are carried out under simulated combat conditions. Now, in the case of the field artillery, rather than a subjective evaluation by the umpires of successful or unsuccessful performance, the unit is required to demonstrate certain basic capabilities. Thus, the primary determinant of success is the unit's ability to get the rounds on target within certain specified time limits after receipt of the mission. In each case, the types of possible missions, and their associated minimum performance requirements, are prescribed by official documents, thus further removing the testing procedure from the vagaries of different umpire opinions as to what may constitute adequate combat performance.

At this point, the reader may be disposed to interject that the procedure outlined above may be very good for obtaining overall mission response times and the associated mean, variance and other distributional information, but that this does not give us any clue as to the specific distributions to be used in modeling the sub-stages of our fire support process. And, indeed, this is just the case if we base our data gathering merely on the written guidelines for conducting the test. Fortunately for Operations Researchers, however, we must go beyond the officially prescribed procedures to see how they are implemented during





the actual conduct of the test. As it so turns out, due to range requirements and various safety restrictions that we impose on training exercises, it is not possible to conduct this simulated combat exercise under true combat conditions. Thus, in actual combat we might be able to say: the battery received the mission at 1700 and reported completion at 1704, therefore total mission time is 4 minutes. However, due to those safety restrictions and other requirements noted above, this is not the case in testing. Instead, we find that the firing process is often interrupted for such reasons as computational checks to insure the weapons are aimed properly (since the artillery is employed out of sight of the target) and also to insure that the correct settings are made on the weapons themselves, both for safety as well as testing purposes, among other reasons. Since these delays would not normally be a part of the combat procedures, where the requirements for rapid fire support would over-ride many of these extra safety checks imposed in peacetime training, it is unfair to count these delays against the unit's performance time. In addition, since these delays are also involved with human responses themselves, they have their own features of variability, so that we cannot simply subtract a constant safety time from each total mission time.

Therefore, in order to avoid this entire problem and still measure the combat capability of the unit relatively accurately and efficiently, the umpires are provided with stop watches and specific personnel are assigned to monitor and time the unit's operations in each of the three sub-stages we identified in our queueing model: the Fire Direction Center, the Firing Battery, and the Forward Observer in his observation post. While many test teams merely record the individual delay times and subtract these from the total mission time, others follow the more direct procedure of measuring the unit's response and, in some rare



cases, the test teams measure both times as a check on the computation of total mission time. Nevertheless, regardless of the procedure used, which is currently at the discretion of the test team, the necessary data collectors are in position with their stop watches, in essence gathering the data we need for our models. It would thus be quite simple to require that the umpires measure the unit's response time (or both the response times and safety times) and report the individual events as they currently do. However, instead of discarding this "component" data after calculating the mission time for the test, we should require that the responses be forwarded to the appropriate agency or modeling group while still divided into the three classes of response times we need for our models. It will then be possible, after sufficient data has accumulated from the testing of the various tactical units, to make an elementary distributional analysis of the individual sub-stage responses to determine not only the mean, but the variance and appropriate distribution as well. With this information, we can then proceed to our formulas obtained from the M/G/1 queueing model to determine just where on our randomness scale we are with respect to true response. In the event that we are still close to the deterministic solution, we can then make our choice of models based on the least cost criterion. On the other hand, should we find that we are not obtaining distributions consistent with those assumed in the models, and that there is more randomness in our processes than is currently being portrayed in our models, we can then proceed to modify our simulations to reflect the true state of nature if the differences justify the amount of modification involved. Thus, with our queueing approach, we have an easily implemented, low cost technique, for deciding whether to continue to develop stochastic models, or to make distributional modifications, without the need to make the modeling changes first and then evaluate the results of our



changes.

### C. RECOMMENDATIONS FOR FURTHER STUDY

While we have reached some rather important conclusions from our study of the fire support process, there is still plenty of room for further investigation to gather the essentially no-cost information about the service distributions from actual process, and that arises naturally from our queueing formulation, would be n. The first area of concern that might provide more insight into the combat field operations. Having then obtained actual knowledge of the overall service distribution from the convolution of the sub-stages, we can then go back to our original G/G/1 model, fix the service distribution to the known case, and determine what the effects of different arrival processes are. However, as noted earlier, the dependence of the arrival distribution on the enemy's force mix and tactical dispositions makes it highly unlikely that we would be able to measure the true state of nature. On the other hand, such an analysis might still provide us with enough general guidelines about battlefield activity as to enable us to identify significant operational trends if certain situations occur in practice as well as theory.

The second area that provides a promise of fruitful research is concerned with evaluating the effects of assigning different priorities to targets, in order to obtain better firing disciplines. This point was also discussed earlier when we considered the importance of mobile versus stationary targets, and their associated maximum delay times for initiating fires. As a direct result of the current study, we can see that our understanding of delay times is causing us to account for a







greater number of target (both mobile and stationary) as casualties, than would be the case in actual combat. While we would certainly hope that the data gathering plan suggested in the last section would provide us with more accurate measures of mobile casualties (due to the delays that affect our ability to fire on them) we would also like to test the effects of various decision rules currently played in various models. For example, the AMMO RATES methodology, which we examined in some detail, uses a procedure such that if a mobile target cannot be taken under fire within a specified time frame, it is to be discarded and ignored as having passed out of the system unless it is reacquired at a later time. Since these time limits, whether based on higher resolution stochastic models or pure military experience, are somewhat suspect as regards the validity of the waiting period, it would be desirable to test different waiting cutoffs and priority assignment procedures. Unfortunately, this would require the use of a simulation, even using a queueing methodological approach, because the treatment of different priorities soon takes us out of the realm of closed-form solutions. However, in this regard, I would like to mention that a simulation of our queueing model, using the SIMSCRIPT Programming language, has been built and debugged by the author. A copy of the program can be obtained on request from Professor James G. Taylor of the Naval Postgraduate School. The simulation, in addition to recording data regarding the factors affecting the timeliness of fires, also has the capability of recording various ammunition expenditures due to the firing process and can easily be adapted to allow for specific lethality subroutines if desired.

#### D. ACKNOWLEDGEMENTS



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## APPENDIX A

### QUEUEING TERMINOLOGY AND NOTATION

#### A. BASIC NOTATION

A convenient shorthand notational system has been developed in queueing theory to facilitate the full specification of queueing disciplines without the necessity of continually writing out all of the significant descriptors of a particular model. The full five-part descriptor is of the form  $A/B/m/K/M$ , where the letters denote the following parameters:

1. A indicates the interarrival distribution
2. B indicates the service time distribution for the system
3. m indicates the number of servers in the system
4. K indicates the system's maximum storage capacity
5. M indicates the system's total population

The last two elements are often dropped when we do not wish to consider such things as bulk arrivals, limiting storage, or finite populations. Thus, the shortest code often used is:  $A/B/m$ , and we assume that the descriptors which are absent take on the value of infinity. For example, the system  $M/M/1$ , perhaps the most common model in



the literature and one of the simplest to analyze, is a single server system with exponential interarrival times and exponential service times which is capable of handling any size queue or customer population. D/M/2/20 is a two-server system with deterministic (constant) interarrival times, exponentially distributed service times, and a storage capacity of 20 customers or queue members, after which any further arrivals are not accepted into the system.

## B. DISTRIBUTIONAL SYMBOLS

The following is a partial list of common distributional symbols used in this shorthand technique:

1. M - exponential distribution
2.  $E^r$  - r-stage Erlang distribution
3.  $H^R$  - Hyper-exponential distribution with R elements
4. D - Deterministic
5. G - General (i.e. not specified)

## C. QUEUEING DISCIPLINES

The following is a partial list of common queueing disciplines, i.e. ways in which the queue or service is organized:

1. FIFO - First in first out, also called First come first served
2. LIFO - Last in first out





3. Priority - Type of service depends on the category or priority of the customer
4. Pre-emptive Priority - Customers with higher priorities take precedence over others in the queue (i.e. they can "butt into line") and/or replace customers already being served



## APPENDIX B

### AN UPPER BOUND FOR THE NORMAL CASE

During the analysis in Chapter VIII, the claim was made that the previously quoted assumptions used in the early DYN-TACS formulation and, most recently, by the CLGP COEA, resulted in an upper bound for the square of the coefficient of variation, which is one of the driving parameters of our queueing model. The reader may recall that the initial assumptions held that each of the individual sub-stage responses was to be modeled using a Normal distribution with a standard deviation equal to 20 percent of the value of the mean, thus:  $\sigma = 0.2 \mu$  for each response distribution. Since each stage of service in our model (FDC, etc.) uses a Normal distribution with these assumptions, and since the reaction time of each stage is independent of the length of time it takes the other stages to function, we have the sum of independent Normal random variables yielding our measure of total service time per mission, which is also Normally distributed with mean response equal to the sum of the sub-stage means and variance equal to the sum of the sub-stage variances, by the reproductive property of the Normal distribution. However, while this service distribution is thus still "Normal", the square of the coefficient of variation of this composite distribution is no longer equal to 0.04. In order to illustrate this point, let's examine a simple 2-stage process where  $a$  is the mean response of the first stage,  $b$  is the mean response of the second stage, and each stage has a standard deviation equal to 20 percent of its mean. Then, for  $a$  and  $b$  individually,



the square of the coefficient of variation is:

$$C_a^2 = \frac{(0.2a)^2}{a^2} = 0.04 \quad \text{and} \quad C_b^2 = \frac{(0.2b)^2}{b^2} = 0.04$$

However, when we take the total response due to passing through both stages sequentially, we note the following, where the reproductive property of the Normal distribution yields a Normally distributed service response with:

$$E(Svc) = E(\text{Stage 1}) + E(\text{Stage 2}) = a + b$$

$$V(Svc) = V(\text{Stage 1}) + V(\text{Stage 2})$$

$$= 0.04a^2 + 0.04b^2 = 0.04(a^2 + b^2)$$

With this composite process, we now observe:

$$C_{Svc}^2 = \frac{V(Svc)}{[E(Svc)]^2} = \frac{0.04(a^2 + b^2)}{a^2 + b^2 + 2ab}$$

and, since we are dealing with response times which, by definition, are positive, we note that the denominator is greater than the sum of the squares of  $a$  and  $b$ , thereby reducing the value of the square of the coefficient of variation from 0.04 to a smaller value. Thus, the more stages we add, the faster we reduce the value of the square of the coefficient of variation from 0.04 towards zero, since the more stages we have, the greater the number of positive cross-product terms ( $ab$ ,  $ac$ ,  $bc$ , etc.) that will appear in the denominator. Thus, it is a simple matter of using the techniques of mathematical induction to prove that, regardless of the mean values and the number of





sub-stages, this procedure of determining the value of the standard deviation to be used by the individual sub-stage processes will reduce the observed random effects to an even greater degree than that of a single stage using the same basic assumption as noted above. However, this upper bound has a distinct use, since it provides us with a measure of the best performance we could obtain if we applied these same assumptions to the composite service process rather than the individual stages.



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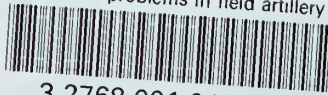
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